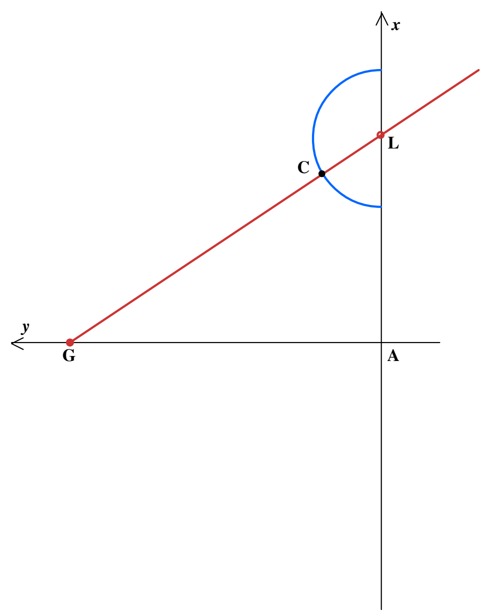
MAT 336.01 Spring 2020 Homework Due April 9

1. Descartes states that the equation $y^2 = cy - \frac{cx}{b}y + ay - ac$ $(*)$ defines a hyperbola. Check this by hand, graphing $(*)$ with $a = 10, b = 3, c = 2$. [One way: divide by y and rearrange to get $x = \frac{b}{c}(c+a) - \frac{b}{c}y - \frac{ba}{y}$, plot the two curves $x_1 = \frac{b}{c}(c+a) - \frac{b}{c}y$ (line) and $x_2 = -\frac{ba}{y}$ (rectangular hyperbola passing through $y = a, x = -b$, $y = b, x = -a$ etc.) and add the two curves graphically: $x = x_1 + x_2$.]

2. Later in that section, Descartes states that if you substitute for the triangle KLN a semi-circle with center L and diameter along the x-axis (as before; see figure) and let C be the intersection of the ruler and the semi-circle, the resulting curve will be "the first conchoid of the ancients." As before, the ruler pivots about the point G, and the ruler is "hinged" to the semicircle at L. Take b as the radius of the circle, and a as before.

   i graphically sketch the conchoid when $b < a$ and when $b > a$. Say $a = 10, b = 5$ and $a = 10, b = 15$.

   ii Imitate Descartes' calculation with KLN to find the equation of the conchoid.



3. Follow Galileo's algorithm to calculate by hand the square root of 1000. Use his approximation with (remainder)/(2 $\times$ integer root) and compare your answer with that from a calculator.