Winter School: Boundary and Singularity in Fluid Mechanics

Student Talks

Schedule

(1) 11:00 – 11:15 Roberto Colombo
(2) 11:15 – 11:30 Yupei Huang
(3) 11:30 – 11:45 Marc Nualart
(4) 11:45 – 12:00 Vıctor Navarro Fernandez

(5) 1:30 – 1:45 Ely Sandine
(6) 1:45 – 2:00 Giorgio Cialdea
(7) 2:00 – 2:15 Gavin Pandya
(8) 2:15 – 2:30 Allison R Byars

(9) 3:00 – 3:15 Filippo Giovagnini
(10) 3:15 – 3:30 Anping Pan
(11) 3:30 – 3:45 Simon Kuang
(12) 3:45 – 4:00 Chanjin You

(13) 4:00 – 4:15 Faiq Raees
(14) 4:15 – 4:30 Hyunwoo Kwon
(15) 4:30 – 4:45 Mustafa Sencer Aydin
(16) 4:45 – 5:00 Jason Zhao

Allison R Byars

Title: Global Dynamics of small data solutions to the Derivative Nonlinear Schrödinger equation.

Abstract: In this talk, we consider the derivative nonlinear Schrödinger (DNLS) equation. While the existence theory has been intensely studied, properties like dispersive estimates for the solutions have not yet been investigated. Here we address this question for the problem with small and localized data, and show that a dispersive estimate for the solution holds globally in time. For the proof of our result we use vector field methods combined with the testing by wave packets method, whose implementation in this problem is novel.

Ely Sandine

Title: Existence of more self-similar implosion profiles for the gravitational Euler-Poisson system.

Abstract: I will discuss an implosion scenario for the equations describing the evolution of an isentropic gas which is compressible, isothermal and self-gravitating. Under the hypotheses of radial symmetry and self-similarity, the equations reduce to a system of ODEs which has been

studied by the astrophysical community using numerical methods. One solution to this system, discovered by Larson and Penston in 1969, was rigorously proved to exist by Guo, Hadžić and Jang. In this talk, I will discuss recent work establishing the existence of a subset of the discrete family of self-similar solutions found numerically by Hunter in 1977. We use methods developed by Collot, Raphaël and Szeftel in the context of energy-supercritical nonlinear heat equations.

Mustafa Sencer Aydin

Title: Uniform bounds and the inviscid limit for the Navier-Stokes equations with Navier boundary conditions

Abstract: We consider the vanishing viscosity problem for solutions of the Navier-Stokes equations with Navier boundary conditions in the half-space. We lower the currently known conormal regularity needed to establish that the inviscid limit holds. Our requirement for the Lipschitz initial data is that the first five conormal derivatives are bounded along with two for the gradient. The requirement on five conormal derivatives can be reduced to four in the case of a favorable sign of the friction coefficient. In addition, we establish a new class of data leading to the local existence and uniqueness for the Euler equations in the half-space or a channel with a condition on the conormal space but without conormal requirements on the gradient.

Víctor Navarro Fernández

Title: Mixing and ideal dynamo with randomized ABC flows

Abstract: In this work we consider the Lagrangian properties of a random version of the Arnold-Beltrami-Childress (ABC) vector fields in a three-dimensional periodic box. We prove that the associated flow map possesses a positive top Lyapunov exponent and its associated one-point, two-point and projective Markov chains are geometrically ergodic. For a passive scalar, it follows that such a velocity is a space-time smooth exponentially mixing field, uniformly in the diffusivity coefficient. For a passive vector, it provides an example of a universal ideal (i.e. non-diffusive) kinematic dynamo. This is a joint work with M. Coti Zelati (Imperial College London).

Hyunwoo Kwon

Title: Nonstationary Stokes equations on a domain with curved boundary under slip boundary conditions

Abstract: We consider nonstationary Stokes equations in nondivergence form with variable viscosity coefficients and Navier slip boundary conditions with slip coefficient α in a domain Ω. First, under the assumption that α is sufficiently smooth, we establish a priori local regularity estimates for solutions near a curved portion of the domain boundary. Second, when α depends on the boundary curvature, we derive local boundary estimates for the Hessians of the solutions, where the right-hand side does not involve the pressure. Notably, our results are new even if the viscosity coefficients are constant. The talk is based on the joint work with Hongjie Dong (Brown University)

Faiq Raees

Title: On The Hydrostatic Approximation of Navier-Stokes-Maxwell System with 2D Electronic Fields

Abstract: In this talk, we establish the local well-posedness of a scaled anisotropic Navier-Stokes-Maxwell system in a two-dimensional striped domain with a transverse magnetic field around $(0, 0, 1)$ in Gevrey-2 class. We also justify the limit from the scaled anisotropic equations to the associated hydrostatic system and obtain the precise convergence rate. Then, we establish the global stability of the state $(0, 0, 1)$ and show that small perturbations decay exponentially. Finally, we discuss the optimality of the Gevrey-2 regularity by proving the solution to a related toy model around shear flow $(y(1-y), 0, 0)$ with some initial data (ζ, ζ^1) grows exponentially.

Chanjin You

Title: Phase mixing estimates for the nonlinear Hartree equation of infinite rank

Abstract: The Hartree equation is a mean-field model that describes many-body quantum systems. In this talk, I will present phase mixing estimates for the density associated with the Hartree equation near certain translation-invariant equilibria. These estimates are important to understand the long-time behavior of quantum particles in the context of quantum Landau damping. The proof relies on the pointwise estimate of the Green function associated with the linearized operator and a nonlinear iterative scheme.

Filippo Giovagnini

Title: A uniform point vortex approximation for the solution of the two-dimensional Navier Stokes equation with transport noise

Abstract: We study a model of interacting particles represented by a system of N stochastic differential equations. We establish that the (mollified) empirical distribution of the system converges uniformly with respect to both time and spatial variables to the solution of the two-dimensional Navier–Stokes equation with transport noise. The proofs are based on a semigroup approach.

Jason Zhao

Title: Local well-posedness for dispersive equations with bounded data

Abstract: Given sufficiently regular data without decay assumptions at infinity, we prove local well-posedness for semi-linear dispersive equations, such as the NLS and gKdV equations. Our approach revisits the classical energy method by applying it to a class of local Sobolev spaces adapted to the dispersion relation. As an application, we show that if in addition the initial data is spatially almost periodic, then the solution remains spatially almost periodic.

Yupei Huang

Title: Classification of the analytic steady states of 2D Euler equation

Abstract: Classification of the steady states for 2D Euler equation is a classical topic in fluid mechanics. In this talk, I will talk about our results concerning the rigidity of the analytic steady states in bounded simply-connected domains. We prove the stream functions for the steady state are either radial function or solutions to semi-linear elliptic equations. The key idea is deriving an over-determined elliptic system when ψ fails to solve a global semi-elliptic equation. If time permits, I will discuss an application of our classification result. This is the joint work with Tarek Elgindi, Ayman Said and Chunjing Xie.

Giorgio Cialdea

Title: Vorticity blowup in 2D compressible Euler equations

Abstract: We show finite-time vorticity blowup for smooth solutions of the 2D compressible Euler equations with smooth, localized, and non-vacuous initial data. The vorticity blowup occurs at the time of the first singularity, and is accompanied by an axisymmetric implosion in which the swirl velocity enjoys full stability, as opposed to finite co-dimension stability. This is a joint work with Jiajie Chen, Steve Shkoller and Vlad Vicol.

Marc Nualart

Title: Streamline Geometries of Steady Euler Flows

Abstract: Steady states of the two-dimensional Euler equations generally come in infinite-dimensional families and play an important role in the long-time dynamics of generic initial data. In this talk we will review several well-known families of steady states, discuss recent results on stationary structures nearby them and finally we will provide a geometric characterization of steady Euler flows in the periodic channel and annulus.

Roberto Colombo

Title: Recent developments in the theory of transport equations with rough vector fields.

Abstract: We consider the problem of well-posedness for the transport equation with Sobolev vector fields advecting unbounded densities. After the celebrated work of Di Perna and Lions, in the very last few years, many achievements have been made in determining the sharp relation between the Sobolev exponent of the field and the integrability of the density yielding to uniqueness of the PDE. In this talk we will be interested in the use of the method of convex integration to construct non-unique solutions of the transport equation, pioneered by Modena and Szekelyhidi and continued by Brué, Colombo and De Lellis. In a joint work with Maria Colombo and Anuj Kumar, we propose a new scheme which, leveraging a different use of intermittency, allows to extend the range of exponents for non-uniqueness previously obtained by convex integration techniques.

Anping Pan

Title: Lagrangian-Eulerian formulation and Variational Principles for some Hydrodynamic PDEs.

Abstract: The classical geodesic formulation of incompressible Euler equation due to Arnold and Ebin-Marsden uncovered variational nature of fluid PDEs, leading to variational formulation of many other inviscid hydrodynamic models and their stochastic counterparts. In this talk I'll discuss a stochastic Hamilton-Pontryagin type variational principle, which incorporates many hydrodynamic models, both inviscid and viscous, as critical point equations of some augmented Lagrangian functional. We also show equivalence between critical point equations of our functional and a fixed point problem with mixture of Lagrangian and Eulerian variables, which generalizes the stochastic Lagrangian formulation of Navier-Stokes equation by Constantin-Iyer and leads to a self-contained Lagrangian proof of local well-posedness of strong solutions using Picard Iteration.

Gavin Pandya

Title: Generalized Riemann variables for shock formation in compressible Euler

Abstract: The compressible Euler equations are known to form shock waves from smooth initial data, but prior to the development of such a singularities, a spacetime codimension-2 manifold is formed, consisting of all spacetime points at which the gradient of the Euler solution has first be-come infinite. Specifically, from smooth and compressive data, the Euler solution first forms a $C^{1/3}$ solution, with the cusp forming along this pre-shock manifold of gradient catastrophes. It is directly from the pre-shock set, that the discontinuous shock surface develops. By a transformation of spacetime which flattens the pre-shock manifold onto a horizontal time-slice, and a system of "differentiated Riemann variables", we are able to smoothly and stably capture the formation of a singularity at the preshock.

Simon Kuang

Title: Nonlinear high-pass phenomena in ODEs

Abstract: I consider nonlinear ODE-based open systems whose properties of asymptotic convergence and regularity depend only on high derivatives of the forcing function. These open systems generalize bounded linear operators that annihilate polynomials of a given degree. Application to integral action control with nonlinear feedback. Key words: dissipativity (in the sense of Willems), uniform integrability in t, convex conjugacy.