(1) Find the point on $x^2 - z^2 = 1$ closest to the origin.

(2) Prove that the arithmetic mean is always greater than or equal to the geometric mean:
$$(x_1 x_2 \cdots x_n)^{1/n} \leq \frac{x_1 + x_2 + \cdots + x_n}{n}$$
where $x_i \geq 0$. *Hint: Maximize $f(y_1, \cdots, y_n) = y_1 \cdots y_n$ subject to $g(y_1, \cdots, y_n) = y_1 + \cdots + y_n = 1$ where $y_i \geq 0$.*

(3) A light ray travels from point A to point B crossing a boundary between two media (say the interface is along $\{y = 0\}$). In the first medium its speed is v_1, and in the second it is v_2. Show that the trip is made in minimum time when Snell's law holds:
$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$
where θ_1 is the angle between the ray of light in the first medium with the normal to the interface (the vector $e_2 = (0, 1)$) and θ_2 be the angle in the second medium.

(4) Find the shortest distances between the points of the line $x + y = 8$ and the ellipse $x^2 + 2y^2 = 6$. *Hint: Pick a point (u, v) on the line and a point (x, y) on the ellipse, and minimize the distance between the two points.*

(5) [1] A die shows k with probability p_k for $k = 1, 2, \cdots, 6$. Consider the vector $\vec{p} = (p_1, p_2, p_3, p_4, p_5, p_6)$, where $\sum_{i=1}^{6} p_i = 1$. This vector is called the probability distribution of the die. The **entropy** of the die is defined as
$$f(\vec{p}) = -\sum_{i=1}^{6} p_i \log p_i.$$
Find the distributions $\vec{p}$ that minimizes and maximizes the entropy.

(**Extra**) [1] *Marsden & Tromba*: §3.2 #6, 9; §3.3 #12, 23, 29, 42; §3.4 #4, 12, 30.

[1] Not to appear on quiz.