(1) Set $\vec{r}(x, y, z) = (x, y, z)$, and $r = \sqrt{x^2 + y^2 + z^2} = \|\vec{r}\|$.

(a) Show that $\nabla \cdot (r^n \vec{r}) = (n+3)r^n$. In particular, $\nabla \cdot (\vec{r}/r^3) = 0$.

(b) Show that $\nabla \times (r^n \vec{r}) = \vec{0}$.

(c) Compute the divergence and curl of

$$\vec{v}(x, y, z) = (A \sin z + C \cos y, B \sin x + A \cos z, C \sin y + B \cos x).$$

(2) The partial differential equation for a smooth function $f(x, t)$

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

models the displacement of a 1-dimensional vibrating string from its equilibrium position with wave velocity c. Set $u = x + ct$ and $v = x - ct$ so that $(x, t) = \left(\frac{u+v}{2}, \frac{u-v}{2c}\right)$. Define

$$F(u, v) = f(x, t) = f\left(\frac{u+v}{2}, \frac{u-v}{2c}\right).$$

Show that

$$\frac{\partial^2 F}{\partial u \partial v} = 0.$$

Deduce that $F(u, v) = g(u) + h(v)$ for some differentiable functions g and h over any rectangle in the uv-plane.

(3) Classify the critical points of $f(x, y) = \sin(xy)$.

(4) Calculate $1.98^{2.01}$ using a linear approximation, and by calculator. This is the output of the function $f(x, y) = x^y$ with $x = 1.98$ and $y = 2.01$. Is the value of f more sensitive to small changes in x or in y?

(5) Find the quadratic Taylor approximation for $f(x, y) = \sin(x^2 + y) + y$ near $P = (0, \pi)$.

(6) Consider the function $f(x, y) = \ln(\sqrt{x^2 + y^2} + y)$.

(a) Determine the domain of f and sketch it in the xy-plane.

(b) What is the linearization of f at $(3, -4)$?