(1) (a) Consider the Archimedean spiral $\vec{r}(t) := (x(t), y(t)) = (t\cos(t), t\sin(t))$ and the scalar function given by $f(x, y) = x + 2y$. Find the velocity of this parametrization. Compute the gradient of f. Find the result, $g(t) = f(x(t), y(t))$, of evaluating the function f along the curve. Take the derivative of g and verify that it is indeed the dot product of the velocity and the gradient evaluated on the curve.

(b) Find an example of a non-constant function that is constant on the Archimedean spiral.

(c) Find an example of a curve on which the function $f(x, y)$ is constant.

(2) Describe and sketch the behavior, as c varies, of the level curves $f(x, y) = c$ for each of these functions
(a) $f(x, y) = x^2 + y^2 + 1$,
(b) $f(x, y) = 1 - x^2 - y^2$,
(c) $f(x, y) = x^3 - x$,
(d) $f(x, y) = x^2 - y^2$,
(e) $f(x, y) = \max\{|x|, |y|\}$.

(3) The matrix $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ acts on a vector $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ by rotating it θ degrees counterclockwise. For example,

$$R_{\pi/2}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad R_{\pi/2}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

(a) Find a function whose level curves are the same as in #2(e), but rotated $\pi/4$ counterclockwise.
(b) Sketch the level curves of $g(x, y) = x^2 - y^2$ and $h(x, y) = xy$ and explain how the two graphs of g and h are related to each other.

(4) Find $\partial f/\partial x$, $\partial f/\partial y$ if
- $f(x, y) = xy$
- $f(x, y) = e^{xy}$
- $f(x, y) = x\cos x \cos y$
- $f(x, y) = (x^2 + y^2)\log(x^2 + y^2)$.

(5) Suppose that the temperature at the point (x, y, z) in space is $T(x, y, z) = x^2 + y^2 + z^2$. Let a particle follow the helix $\vec{r}(t) = (\cos t, \sin t, t)$ and let $T(t)$ be its temperature at time t.
- What is $T'(t)$?
- Find an approximate value for the temperature at $t = \frac{\pi}{2} + 0.01$.

(6) Compute the directional derivative of f in the given directions $\vec{v}$ at the given points P.
- $f(x, y, z) = xy^2 + y^2z^3 + z^3x$, $P = (4, -2, 1)$, $\vec{v} = \frac{1}{\sqrt{14}}(\vec{i} + 3\vec{j} + 2\vec{k})$
- $f(x, y, z) = e^{-z}\sin(x)\sin(y)$, $P = (\pi, \frac{\pi}{2}, 0)$, $\vec{v} = \frac{12}{13}\vec{i} + \frac{3}{13}\vec{j} + \frac{4}{13}\vec{k}$.

(7) In electrostatics, the force $\vec{P}$ of attraction between two particles of opposite charge is given by $\vec{P} = k\frac{\vec{r}}{\|\vec{r}\|^3}$ (Coulomb's law), where k is a constant and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. Show that $\vec{P}$ is the gradient of $f = -\frac{k}{\|\vec{r}\|}$.

(8) * Fix two points p_1 and p_2 in $\mathbb{R}^2$. Let

$$d(\vec{x}) = \|\vec{x} - \vec{p}_1\| + \|\vec{x} - \vec{p}_2\|.$$

(a) What are the level lines of the function d?

(b) Show that the normal vector to a level set of d at some point p has the following property: it bisects the the angle between the vectors which point from p to p_1 and to p_2 respectively.

(**Extra**) * *Marsden & Tromba*: §2.3 #9(c), 14, 18, 22, §2.4: #18, 25; §2.5: #3(d), 7, 22; §2.6: #8(a).

*Not to appear on quiz.