(1) Find the volume of the region sitting above the xy–plane bounded by $x^2+y^2=1$, $x=z$ and $z=0$. This region is known as the *hoof of Archimedes*.

(2) Evaluate the following integrals using polar coordinates

(a) $I=\int_0^1\int_0^{\sqrt{1-y^2}}(x^2+y^2)dxdy$,

(b) $I=\int_{\sqrt{2}}^2\int_{\sqrt{4-y^2}}^y dxdy$,

(c) $I=\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{2}{(1+x^2+y^2)^2}dydx$.

(3) Use Green's theorem to compute $\oint_C \vec{F}\cdot d\vec{r}$ if $\vec{F}=(\arctan(y/x),\log(x^2+y^2))$ along the curve C given by the boundary of the region defined by the polar inequalities $1\le r\le 2$ and $0\le\theta\le\pi/2$, oriented counterclockwise.

(4) Find the work done by the force $\vec{F}=(4x-2y,2x-4y)$ on a particle going counterclockwise around the circle C: $(x-2)^2+(y-2)^2=4$.

(5) Show that the value of $\oint_C xy^2dx+(x^2y+2x)dy$ around any square C depends only on the area enclosed and not on its location in the plane.

(6) [1] Consider the function $f(\vec{x})=\|x\|^2$ on $\mathbb{R}^3$. It has a unique critical point at the origin $\vec{x}=0$, which is a global minimum. Find a surface in $\mathbb{R}^3$ described by $f(\vec{x})=0$ such that, restricted to this surface, the function has a unique critical point which is a saddle.

[1] Not to appear on the quiz.