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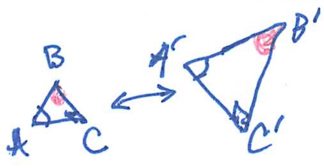
TRIG:

SOME GEOMETRY FIRST.

TWO TRIANGLES ARE <u>SIMILAR</u> IF THERE IS AN ISOMETRY + DILATION WHICH MAPS ONE TO THE OTHER.

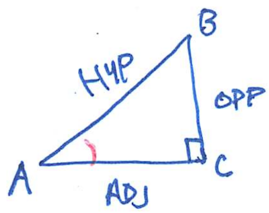


IE, ~~ALL~~ CORRESPONDING ANGLES AGREE AND CORR. SIDES IN PROPOTION

$\measuredangle A = \measuredangle A'$, $\measuredangle B = \measuredangle B'$, $\measuredangle C = \measuredangle C'$, AND $\dfrac{|AB|}{|A'B'|} = \dfrac{|BC|}{|B'C'|} = \dfrac{|AC|}{|A'C'|}$

<u>TWO</u> EQUAL ANGLES SUFFICE (SINCE SUM OF ANGLES IS $180°$).

~~GIVEN~~ THIS MEANS GIVEN ANY RIGHT TRIANGLE, KNOWING ONE OTHER ANGLE ~~GIVES~~ FIX RATIO OF SIDE LENGTHS.



SO WE CAN DEFINE A FUNCTION ON ANGLES WHICH IS THE RATIO OF THE LEGTH OPPOSITE THAT ANGLE TO THAT OF THE HYPOTENEUSE (OPP. THE RIGHT)

$$\boxed{\sin(\measuredangle A) = \frac{|BC|}{|AB|} = \frac{|\text{OPPOSITE}|}{|\text{HYPOTENUSE}|}}$$

THIS ISN'T A TRIANGLE IF $\measuredangle A = 0$ OR $\measuredangle A = 90°$

BUT IT IS EASY TO SHOW

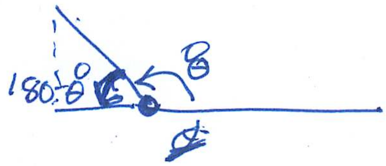
$\lim_{\measuredangle A \to 0°} \sin(\measuredangle A) = 0$ $\qquad \lim_{\measuredangle A \to 90°} \sin(\measuredangle A) = 1$

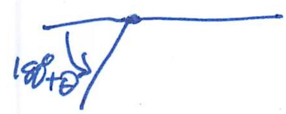
ANALAGOUSLY, CAN DEFINE OTHER RATIONS $\cos(\measuredangle A) \left(= \sin(\measuredangle B)\right)$

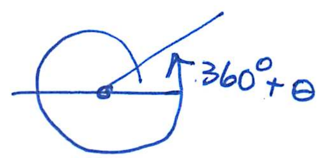
AND $\tan(\measuredangle A) = \dfrac{\sin(\measuredangle A)}{\cos(\measuredangle A)}$

NOW WE EXTEND THIS NOTION TO ALL OF $\mathbb{R}$ IN A NATURAL WAY.

FIRST, NOTICE THAT ~~THERE ARE TWO~~ WE CAN MAKE SENSE OF ANGLES $\geq 90^\circ$

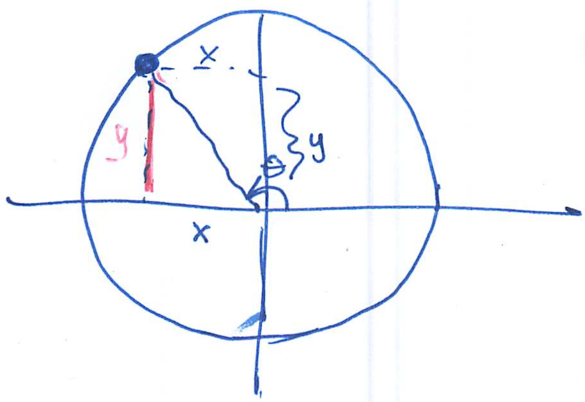






ETC

NOW FIX A CIRCLE OF RADIUS 1:



ANY POINT ON THAT CIRCLE HAS COORDS (X & Y) AND AN ASSOCIATED TRIANGLE WITH AN ANGLE AT THE ORIGIN $0° \leq \theta \leq 360°$

NOTE THAT FOR $0 \leq \theta \leq 90°$, $x = \cos\theta$, $y = \sin\theta$.

BUT OF COURSE THERE ARE X & Y VALUES FOR $0 \leq \theta \leq 360°$

BUT THEN WE EXTEND TO $\mathbb{R}$ BY MAKING IT PERIODIC, WITH THE INTERPRETATION THAT WE ARE KEEPING TRACK OF THE NUMBER OF TURNS

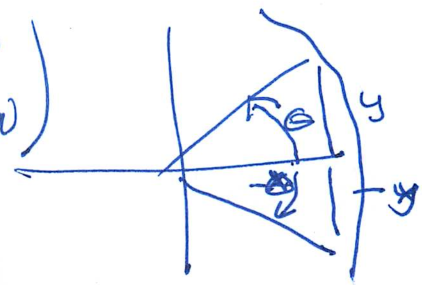
THIS GIVES US SOME IDENTITIES:

$$\sin(\theta) = \sin(\theta + k\cdot 360^\circ) \qquad \forall k \in \mathbb{N}.$$
$$\cos(\theta) = \cos(\theta + k\cdot 360^\circ)$$

BUT ALSO

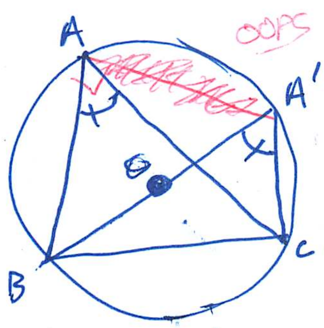
$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$

(SIN IS ODD
COS IS EVEN)



$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$ ← PYTHAGOREAN THM.

LAW OF SINES (FROM GEOMETR)



$$\frac{\sin(A)}{|BC|} = \frac{\sin(B)}{|AC|} = \frac{\sin(C)}{|AB|} = \frac{1}{2r}$$

(r = CIRCUMRADIUS)

[FROM GEOMETRY $\measuredangle A = \measuredangle A'$
CUZ SUBTEND SAME ARC]

~~SEE~~ NOW $\Delta A'BC$ IS RIGHT (w $\measuredangle C = 90^\circ$)

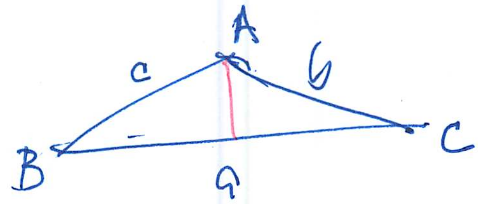
SO $\sin(A) = \sin(A') = \dfrac{|BC|}{|BA'|}$

$$\Rightarrow \frac{\sin A}{|BC|} = \frac{1}{|BA'|}$$

REST FOLLOWS...

LAW OF COSINES

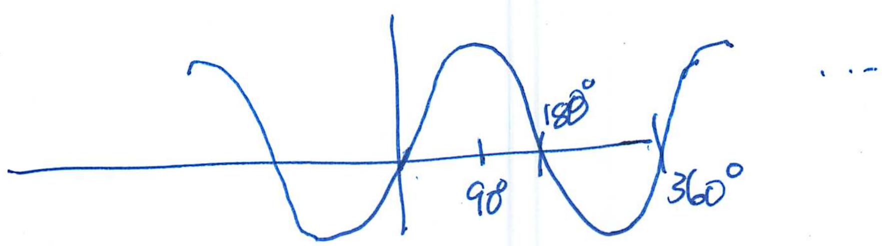
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$



PROOF COMES TO CHOPPING INTO APPROPRIATE RIGHT TRIANGLES. THERE ARE A COUPLE OF CASES.

USING GEOMETRY, WE CAN WORK OUT SPECIAL CASES, EG $\sin 45° = \frac{1}{\sqrt{2}} = \cos 45°$

INDEED, GRAPHING WE SEE



ADDITION FORMULAE

$$\sin(s+t) = \sin(s)\cdot\cos(t) + \cos(s)\sin(t)$$
$$\cos(s+t) = \cos(s)\cos(t) + \sin(s)\sin(t)$$

IF WE GET ONE, THE OTHER FOLLOWS ...

TOO MUCH GEOMETRY + ALGEBRA FOR NOW.

RADIANS. - NOTHING WRONG WITH DEGREES BUT IT IS AN ARBITRARY CHOICE.

COMPARING LENGTHS (RADIANS) IS ~~[illegible]~~ INTRINSIC.

BUT ALSO BETTER FOR CALCULUS.

IN DEGREES $\frac{d}{dx}\sin x = \frac{\pi}{180}\cos x$, JUST UGLY.

RELATION W/ COMPLEX #'S ?

DE MOIVRE (1720s) FOR INTEGERS n

$$(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx)$$

EULER: (1740s) $e^{ix} = \cos x + i \sin x.$

HERE, LETS USE TAYLOR SERIES

$$e^{z} = 1 + z + \frac{z^2}{2} + \frac{z^3}{3!} + \frac{z^4}{4!} + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

GEOMETRICALLY, USING TRIG CAN SHOW

MULT OF COMPLEX NUMBERS $\Leftrightarrow$ ADD ANGLES

MULT LENGTHS.