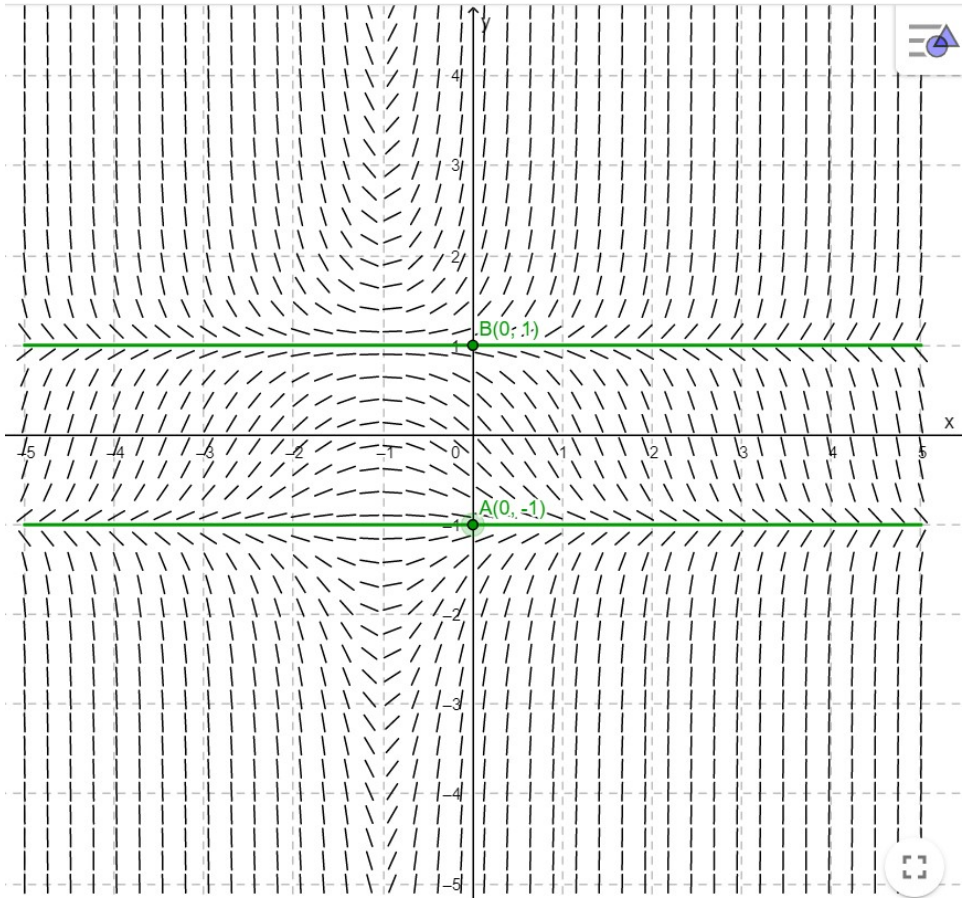

EXERCISE ONE This slope field with two equilibrium solutions corresponds to one of the following differential equation:

1. $y' = 1 - y^2 + x^2$;
2. $y' = (y-1)(1-y)(1+x^3)$;
3. $y' = xy^2 - x + y^2 - 1$.

Which one is it? Consider the particular solution of this equation with initial value $y(0) = 0$; use the slope field to determine whether $y(1) > 0$ or $y(1) < 0$. Please justify your answers.

Let us see whether the equation $y' = 1 - y^2 + x^2$ has equilibrium solutions. Suppose that y is an equilibrium solution. Then $1 - y^2 + x^2 = y' = 0$. This means $y^2 = 1 + x^2$. In particular y is not a constant function (it depends on x), contradiction. Therefore the first equation does not have any equilibrium solution and does not correspond to the slope field.

Consider the second equation $y' = (y-1)(1-y)(1+x^3)$. At the point with coordinates $(0,2)$, we have a slope $(2-1)(1-2)(1+0^3) = -1$. However on the slope field we have a postive slope at $(0,2)$. Thus the second equation does not correspond to the slope field either.

The slopes at points (x, y) with $0 \leq x \leq 1$ and $-1 < y < 1$ are all negative. This means that the particular solution with initial condition $y(0) = 0$ is strictly decreasing on the interval $[0, 1]$. Therefore $y(1) < y(0) = 0$.