

## LIMITS

ROBERT HOUGH

*Problem 1.* Let  $a$  and  $b$  be positive real numbers. Prove that  $\lim_{n \rightarrow \infty} (a^n + b^n)^{1/n}$  is the larger of  $a$  and  $b$ .

*Problem 2.* Prove that  $\lim_{n \rightarrow \infty} (1 + \frac{1}{2} + \dots + \frac{1}{n} - \log n)$  exists and lies between  $\frac{1}{2}$  and 1.

*Problem 3.* Let  $K$  be a positive real number. Take an arbitrary positive real number  $x_0$  and form the sequence  $x_{n+1} = \frac{1}{2} \left( x_n + \frac{K}{x_n} \right)$ . Show that  $\lim_{n \rightarrow \infty} x_n = \sqrt{K}$ .

*Problem 4.* Let  $x_{n+1} = x_n^2 - 6x_n + 10$ . For what values of  $x_0$  is  $\{x_n\}$  convergent and how does the limit depend on  $x_0$ .

*Problem 5.* Is  $\sqrt{2}$  the limit of a sequence of numbers of the form  $n^{1/3} - m^{1/3}$ ?

*Problem 6.* Let  $x_0 = 1$  and  $x_{n+1} = x_n + 10^{-10^{x_n}}$ . Does  $\lim_n x_n$  exist?

*Problem 7.* Show that the improper integral

$$\lim_{B \rightarrow \infty} \int_0^B \sin(x) \sin(x^2) dx$$

converges.

*Problem 8.* Find

$$\lim_{n \rightarrow \infty} \left[ \frac{1}{n^4} \prod_{i=1}^{2n} (n^2 + i^2)^{1/n} \right].$$

*Problem 9.* Evaluate

$$\lim_{n \rightarrow \infty} \sum_{j=1}^{n^2} \frac{n}{n^2 + j^2}.$$

*Problem 10.* Evaluate  $\prod_{n=2}^{\infty} \frac{n^3-1}{n^3+1}$ .

*Problem 11.* Evaluate

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left( \left\lfloor \frac{2n}{k} \right\rfloor - 2 \left\lfloor \frac{n}{k} \right\rfloor \right).$$