

PROBABILITY

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Problem 1. Find the probability that in a group of n people, two people have the same birthday. Assume it isn't a leap year.

Problem 2. What is the probability that a permutation of the first n numbers has 1 and 2 in the same cycle.

Problem 3. An unbiased coin is tossed n times. Find a formula, in closed form, for the expected value of $|H - T|$, where H is the number of heads, and T is the number of tails.

Problem 4. Prove the identities

$$\sum_{k=1}^n \frac{1}{(k-1)!} \sum_{i=0}^{n-k} \frac{(-1)^i}{i!} = 1, \quad \sum_{k=1}^n \frac{k}{(k-1)!} \sum_{i=0}^{n-k} \frac{(-1)^i}{i!} = 2.$$

Problem 5. Given the independent events A_1, A_2, \dots, A_n with probabilities p_1, p_2, \dots, p_n , find the probability that an odd number of these events occurs.

Problem 6. Find the probability that in the process of repeatedly flipping a coin, one will encounter a run of 5 heads before one encounters a run of 2 tails.

Problem 7. A coin is tossed n times. What is the probability that two heads will turn up in succession somewhere in the sequence.

Problem 8. What is the probability that the sum of two randomly chosen numbers in the interval $[0, 1]$ have sum that does not exceed 1 and product that does not exceed $\frac{2}{9}$?

Problem 9. What is the probability that three points selected at random on a circle lie on a semicircle?

Problem 10. If a needle of length 1 is dropped at random on a surface ruled with parallel lines at distance 2 apart, what is the probability that the needle will cross one of the lines?