MAT 342, Homework 4 due 9/25

Questions from the textbook Sec. 34: question **1**, Sec. 37 questions **3, 4, 5**.

More questions:

1. (a) Find all values of $\log(1-\sqrt{3}i)$. Find the principal value $\mathrm{Log}(1-\sqrt{3}i)$.

(b) Find all values of $\log(-i-1)$. Find the principal value $\mathrm{Log}(-i-1)$.

Now make a different branch cut, along the positive x-axis. Consider a branch of log such that

$$\log z = \ln r + i\theta, \qquad 0 < \theta < 2\pi, \qquad \text{for } z = re^{i\theta}.$$

Find the value of $\log(-i-1)$ for this branch. Also find the value of $\log(-1)$ for this branch.

(c) Find all the values of $(-1)^i$. Can you find the principal value?

2. (a) Check that $\cos^2 z + \sin^2 z = 1$ for all complex numbers z. Work directly from the definitions of $\sin z$ and $\cos z$ (formulas involving exponential functions.)

(b) Prove the trigonometric identity

$$\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2,$$

as follows, using properties of exponential functions. First use the definition of $\cos z$, express $\cos(z_1 + z_2)$ via exponential functions. You will get a formula with terms like $e^{i(z_1+z_2)}$. Then write the exponent of sum as a product for each such term,

$$e^{i(z_1+z_2)} = e^{iz_1}e^{iz_2}$$

and expand e^{iz_1} in terms of sine and cosine, and similarly e^{iz_2}. If you multiply out the products, put everything into the expression for $\cos(z_1+z_2)$ collect terms and simplify, you should get the desired identity.

Optional assignment, for deeper understanding of branches of multivalued functions: Sec 33 questions **3–6**.