MAT 342, Homework 3 due 9/12

1. For the following functions, use differentiation rules to find the derivative $f'(z)$ if it exists.

(a) $f(z) = \dfrac{(i - z^2)^5}{(1+i)^2}$ (b) $f(z) = \dfrac{(2 + iz)^3}{iz^2}$

2. Use the Cauchy-Riemann equations to determine where the given function f is differentiable, and compute $f'(z)$ at the points z where it exists. (The answer may be that $f'(z)$ exists at all points or doesn't exist at any point.) In the formulas below, $z = x + iy$.

(a) $f(z) = (z^2 + 1)\bar{z}$ (b) $f(z) = e^{-x}e^{-iy}$ (c) $f(z) = e^{\bar{z}}$ (d) $f(z) = x^2 - iy^2$

3. (a) Suppose that $f'(z)$ exists for all z and f takes only real values, i.e. $f(z) \in \mathbb{R}$ for all z.
Find $f'(z)$; explain your answer carefully.

(b) What can you say about the function $f(z)$, then? Justify your answer carefully.
[Assume that the domain of f is $\mathbb{C}$, that is, $f(z)$ is defined for all complex numbers z.]

Questions from the textbook Sec. 20: **3ab, 8a**.

Optional assignment for students who took MAT 319/320 and/or are interested in proofs: Please do Sec 20 questions **4, 6, 7**, Sec 24 question **8ab**. Read the proofs in section 23 to learn why the Cauchy–Riemann equations give a *sufficient* condition for differentiability.