MAT 342, Homework 2 due 9/11

1. Find all solutions of the following equations. Use exponential form of complex number and show all work step-by-step. (Please do not plug in the given data into the formulas for the solutions.)

$$\text{(a) } z^3 = -27 \qquad \text{(b) } z^8 = 16 \qquad \text{(c) } z^5 = 1 - i$$

For each part, plot all the solutions on the complex plane. (Make 3 pictures, one for each part.) Please label all the relevant information such as points and angles.

In part (c), leave your answer in the exponential form. In parts (a) and (b), convert the answers to the form $z = x + iy$.

2. Make the following plots (with explanation) to illustrate how the complex function $f(z) = z^3$ works.

On the z-plane, plot

- the rays $l_1 = \{\operatorname{Arg} z = \frac{\pi}{4}\}$ and $l_2 = \{\operatorname{Arg} z = \frac{\pi}{3}\}$;
- the circles c_1 of radius 1 and c_2 of radius $\frac{1}{2}$, both centered at 0;
- the points $z_1 = i$ and $z_2 = -1 + i$.

Find the images $f(l_1)$, $f(l_2)$, $f(c_1)$, $f(c_2)$, $f(z_1)$, $f(z_2)$ for the function $f(z) = z^3$ and plot them on a separate w-plane to illustrate the mapping $w = f(z)$. Explain how you found your answers.

Please work with the exponential form (this is easier). See textbook Section 14 Example 2 for inspiration.

3. Determine whether the following limits exist (with explanation); if they exist, compute them.

$$\text{(a) } \lim_{z \to -i} \frac{z^2 + 1}{(z + i)(\bar{z} + 2)} \qquad \text{(b) } \lim_{z \to i} \frac{z - i}{\bar{z} + i}$$

You can use limits laws (theorems from Sections 16 and 17) or direct arguments such as Section 15 Example 1 and Example 2. Full epsilon-delta arguments are not required. (We'll do some similar examples in class on Monday.)

Questions from the textbook Sec. 18: **1, 5, 10ac**. Informal explanations for question 1 will be accepted (rigorous proofs using definition from Sec. 15 are not required,)

Optional assignment for students who took MAT 319 and/or are interested in proofs: Read sections 15-18 carefully and try understand all the epsilon-delta proofs. Give a rigorous epsilon-delta proof for Sec 18 question **1**. Please also do Sec 18 questions **7, 9**. Most of this is very similar to what you learned in MAT 319.