MAT 342, Homework 1 due 9/4

Questions from the textbook Sec. 3: **1a**, Sec 5: **1a, 5b**, Sec 6: **1d, 2b, 7**.

Please also do the following questions:

1. Let $z = 2 - 3i$ and $w = 3 + i$. Simplify the expression

$$\frac{z + 2w}{1 - z\bar{w}}$$

by writing it in the form $a + bi$ for some $a, b \in \mathbb{R}$.

2. (a) In the complex plane, sketch the curve given by the equation $|z + 3 - 2i| = 6$. Explain your answer.

(b) The curve from part (a) cuts the plane into two regions, the region enclosed by the curve and the unbounded region outside of the curve. For each of the regions, write an inequality describing all points z in that region. (For example, the inequality $|z| < 1$ describes points inside the unit circle.) Explain your answers.

(c) Show that the circle $|z| = 1$ lies entirely in the region enclosed by the curve from (a). To do so, check that the corresponding inequality from part (b) holds whenever $|z| = 1$. You will need to use an important general inequality that we learned in class.

3. (a) Write the complex number $z = 1 - \sqrt{3}i$ in exponential form. What is the argument $arg\, z$ and the modulus $|z|$ of this number? What is the principal value $Argz$ of the argument?

(b) Compute $(1 - \sqrt{3}i)^5$.

(c) Compute $(1 + \sqrt{3}i)^5$. Hint: $1 + \sqrt{3}i$ is closely related to $1 - \sqrt{3}i$, so there's a quick way to get the answer from (b) without doing too much work.

The homework can be submitted on Wednesday morning in class or brought to the grader no later than 5pm on the due date. (There will be an envelope outside of S240A, SL floor in Math Building.)