Lecture 1

Predicates

Our goals

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- **Navigate** through any new mathematical environment.

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- Learn how to **cope with difficulties** in math studies.

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- quantifiers
- structure of a theory
- basic methods of proofs

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In the definition of **proposition** above,
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In our course, the word “proposition” will be used in **exactly** this meaning.

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Let us read the **definition** of a **proposition** again.

What is a proposition?

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Definition. A **proposition** is a sentence that is either true or false.

A sentence is a grammatical unit, containing at least a subject and predicate.

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6) $x>1$ Is it a proposition? It is a sentence, whose truth value depends on x. It is so-called **open sentence** or **predicate**.

Predicates and statements

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Values of variables are taken from some **domains**, which are either **explicitly described** or **silently assumed**.

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A statement without a free variable is a proposition,
a statement with a free variable(s) is a predicate.

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So, a Boolean function is a function in n Boolean variables
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The main purpose is to **build** new **statements**
as **compositions** of old statements **with a Boolean function**.

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Truth table for negation

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A propositional form is described by the **truth table**.

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Even in a mathematical context, it may be **uneasy to recognize** a conjunction under these words.

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Exercise. Construct a truth table for the propositional form "either P or Q" and convince yourself that "either P or Q" has true value only if exactly one of P, Q is true.

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$$P \wedge (\neg (Q \implies R) \vee S)$$

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$P \wedge (\neg (Q \implies R) \vee S)$ is a **non-equivalent** proposition.

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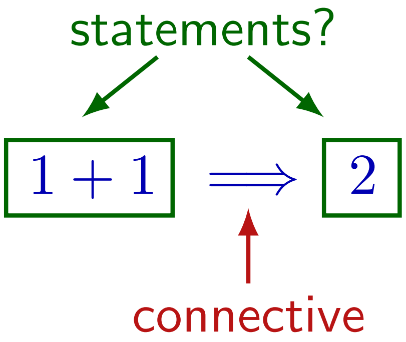

statements?

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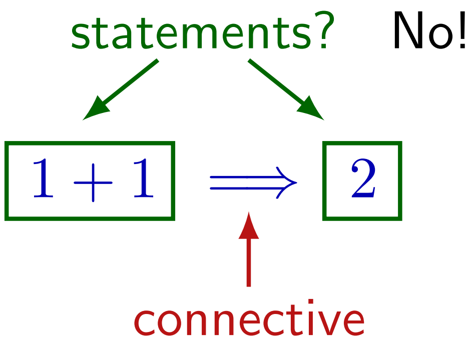

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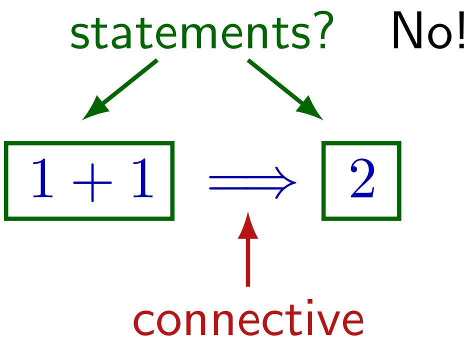

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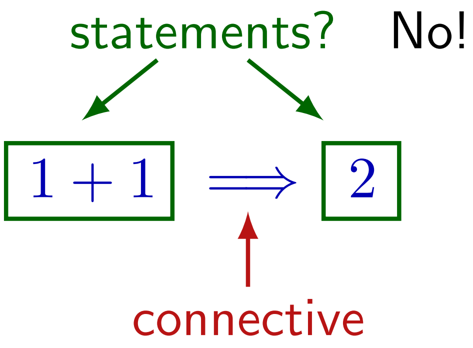

$$\boxed{1 + 1} \implies \boxed{2}$$

- Use appropriate mathematical symbols: $1 + 1 = 2$

Priority of connectives

Example 2. Determine the truth value of the following:

$$\neg(2<1) \implies 1=2 \vee 1+1=2$$

Solution.

$$\underset{\text{T}}{\boxed{\neg\, \underset{\text{F}}{\boxed{(2<1)}}}} \implies \underset{\text{T}}{\boxed{\underset{\text{F}}{\boxed{1=2}} \vee \underset{\text{T}}{\boxed{1+1=2}}}} \quad \text{is } \textcircled{T}$$

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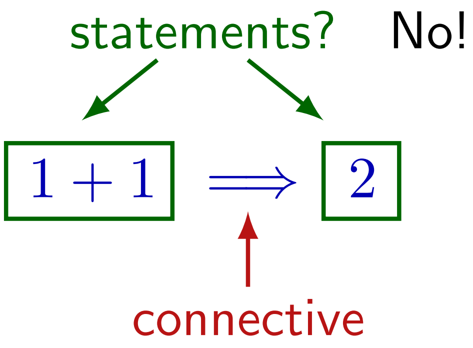

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- Use appropriate mathematical symbols: $1+1=2$
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Examples of compound propositions

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$$\pi = 3.14 \quad \vee \quad e < 3$$

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$\pi = 3.14 \quad \vee \quad e < 3$ T or F?

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Remark. Logical disjunction is **not** exclusive.

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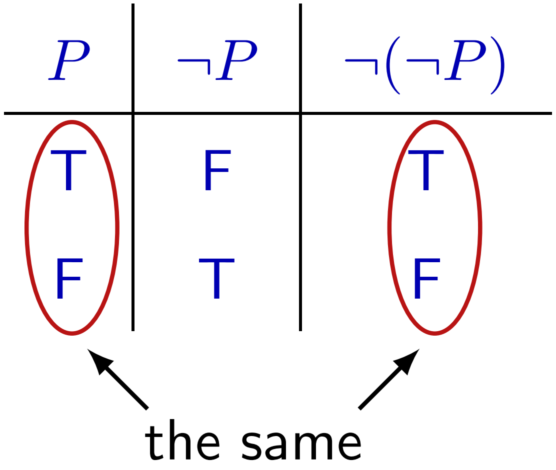

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Tautology and contradiction

Definition. A **tautology** is a propositional form which is **always true.**

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Here the word **always** means **for all values of the arguments**.

Examples. $P \vee (\neg P)$ is a tautology (**Law of excluded middle**)

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For any statements P, Q and R

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When are they true?

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Solution. Choose all rows yielding **T**.
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What are the truth values of them?

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Construct a propositional form in terms of P, Q and $\neg$, $\wedge$, $\vee$ equivalent to R.

Solution. Choose all rows yielding **T**.

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Construct the corresponding conjunctions:

$P \wedge \neg Q$, $\neg P \wedge Q$.

What are the truth values of them?

Conjunction, disjunction and negation

Theorem. Any propositional form is equivalent to a propositional form containing only $\neg$, $\wedge$, $\vee$ as connectives.

A required form can be easily constructed after analyzing the following.

Example. Let P and Q be propositions. Let R be a propositional form taking the following truth values depending on P and Q:

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In order to answer, we need to study b $\Longrightarrow$ and $\Longleftrightarrow$.

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Theorem. Let P and Q be propositions.
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$P \wedge (P \implies Q)$ is equivalent to $P \wedge Q$.

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As soon as one proves P , it can be added to the set of axioms.

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Logic is used in the context of a theory.
Some statements in a theory are accepted to be true (and called **axioms**).
Some statements have been deduced from axioms and have become **theorems**.

What does it mean, to prove a proposition P? Say, let A_1 ,. . . , A_n be the list of axioms.
To prove a proposition P means
to prove the proposition $(A_1 \wedge \cdots \wedge A_n) \implies P$.
As soon as one proves P , it can be added to the set of axioms.
By Modus Ponence, $A \wedge (A \implies P)$ is equivalent to $A \wedge P$.

Conditional sentences

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Examples

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Example 1. Read the sentence

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$$1 > 2 \quad \text{if} \quad 2 > 1$$

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Example 4. Let x be a real number.

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Example 4. Let x be a real number. Let P be the predicate "$x = 1$",

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Example 4. Let x be a real number. Let P be the predicate “$x = 1$”, and Q be the predicate “$x^2 - 3x + 2 = 0$”.

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Example 4. Let x be a real number. Let P be the predicate "$x = 1$", and Q be the predicate "$x^2 - 3x + 2 = 0$".

Determine the truth values of the following statements:

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P if Q

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Determine the truth values of the following statements:

P if Q

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P is sufficient for Q

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Truth value of a predicate

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How can one speak about a **single** truth value?

However,
it may happen that **for all** the values of the variables the sentence holds true.

Then we say that the statement is true.

Otherwise we say that the statement is false.

If we speaks about truth value of a predicate
without mentioning values of variables,
it means that we are speaking truth value for all values of the variables.

Examples

Example 4 (cont.)

Examples

Example 4 (cont.) How are P ($x = 1$) and Q ($x^2 - 3x + 2 = 0$) related?

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Examples

Example 4 (cont.) How are P ($x = 1$) and Q ($x^2 - 3x + 2 = 0$) related?

$x^2 - 3x + 2 = 0 \iff (x-1)(x-2) = 0 \iff x = 1 \lor x = 2$.

Examples

Example 4 (cont.) How are P ($x = 1$) and Q ($x^2 - 3x + 2 = 0$) related?

$x^2 - 3x + 2 = 0 \iff (x-1)(x-2) = 0 \iff x = 1 \vee x = 2$.

Therefore, Q is equivalent to the predicate $x = 1 \vee x = 2$.

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Now we are ready to analyze the given statements:

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Now we are ready to analyze the given statements:

P if Q means

$x = 1 \Longleftarrow x = 1 \vee x = 2$

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$x = 1 \Longleftarrow x = 1 \vee x = 2$, which is false.

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P only if Q

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$x^2 - 3x + 2 = 0 \iff (x-1)(x-2) = 0 \iff x = 1 \lor x = 2$.

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P is sufficient for Q

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P is sufficient for Q means

Examples

Example 4 (cont.) How are P ($x = 1$) and Q ($x^2 - 3x + 2 = 0$) related?

$x^2 - 3x + 2 = 0 \iff (x-1)(x-2) = 0 \iff x = 1 \vee x = 2$.

Therefore, Q is equivalent to the predicate $x = 1 \vee x = 2$.

Now we are ready to analyze the given statements:

P if Q means

$x = 1 \Longleftarrow x = 1 \vee x = 2$, which is false.

P only if Q means

$x = 1 \implies x = 1 \vee x = 2$, which is true.

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Causation VS implication

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The structure “**If** . . . **then** . . .” appears both in **language** and **logic**.

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If it rains , **then** I take an umbrella .

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cause
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If $e < 3$, **then** $\pi > 3$.

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assumption
(not a cause)

conclusion

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conclusion
(not an effect)

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In mathematics, there is **no** causation

Causation VS implication

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cause
(comes first)

effect
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If $\boxed{e < 3}$, **then** $\boxed{\pi > 3}$. It's a **true** statement.

assumption
(not a cause)

conclusion
(not an effect)

In mathematics, there is **no** causation and there are **no** tenses
(past, present, future).

Hidden implications

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Often an implication is **hidden**.

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Example 1. Vertical angles are congruent.

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If a polygon is regular, **then** one can circumscribe a circle around it.

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Example 3. A differentiable function is continuous.

If a function is differentiable, **then** it is continuous.

Converse, contrapositive, inverse

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Let P and Q be statements.

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Theorem. A statement and its contrapositive are equivalent.

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So the implication $P \implies Q$ and its contrapositive $\neg Q \implies \neg P$ are equivalent for any propositions P and Q.

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