Lecture 5

Sets

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How to prove set-theoretic identities
How to prove

Sets

A set and its elements

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In any intellectual activity,
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The **first words** in this language are **set** and **element**.

A **set** is a collection of objects which are called **elements**.
A set **consists** of (and is **defined** by) its elements.

Notations and synonims

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Definition. An **empty set**

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empty box $\neq$ a box containing an empty box.

Subsets

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By definition, $A \subset B \iff \forall x\,(x \in A \implies x \in B)$.

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Example. $A = \{1, 2, 3\}$.

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The signs "$\subset$", "$\subseteq$", "$\supset$" and "$\supseteq$" are called **inclusion** symbols. Commonly $\subset$ and $\subseteq$ are used in the same sense.

By definition, $A \subset B \iff \forall x \, (x \in A \implies x \in B)$.

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Example. $A = \{1, 2, 3\}$.

correct: $1 \in A$ $\{1\} \in A$

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Example. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.

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Theorem (transitivity of inclusion).

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Intersection and Union

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$$A \cap B = \{x \mid x \in A \land x \in B\}$$

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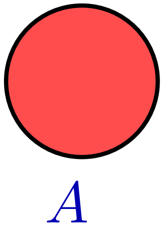

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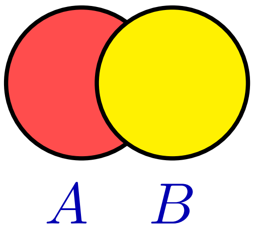

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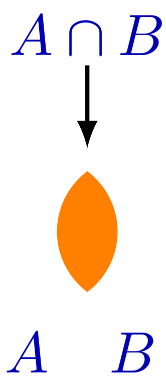

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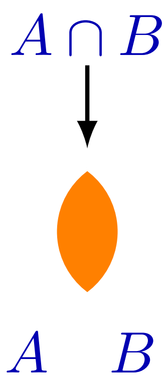

Venn diagram

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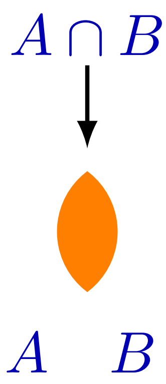

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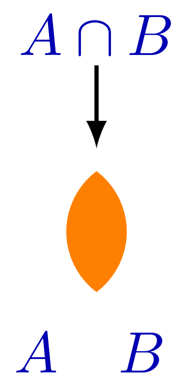

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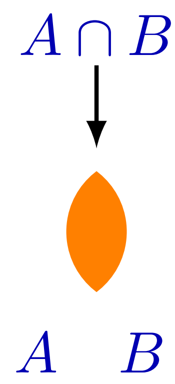

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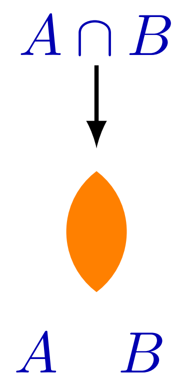

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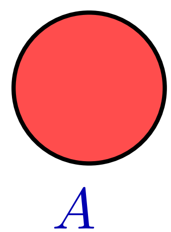

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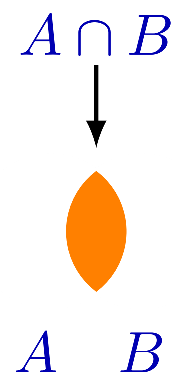

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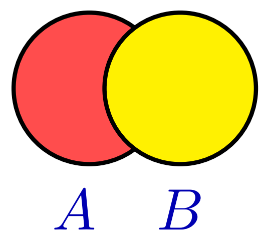

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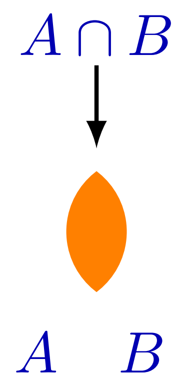

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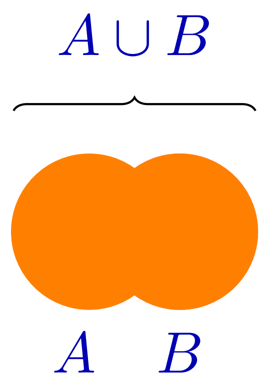

Difference

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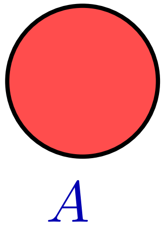

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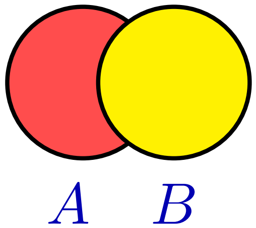

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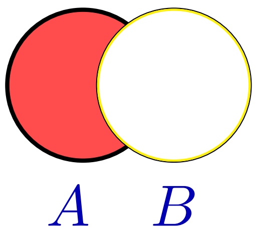

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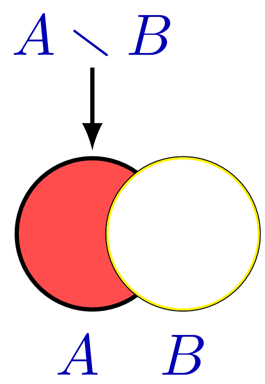

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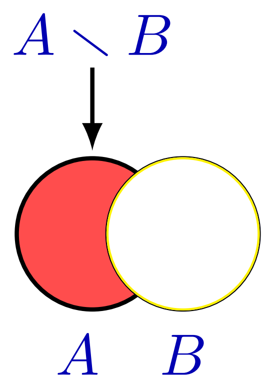

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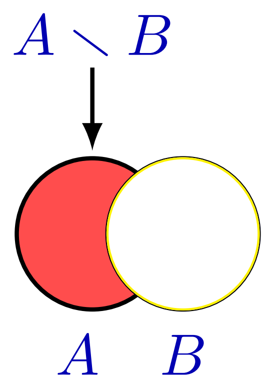

Complement

A^C

Difference

Difference and Complement

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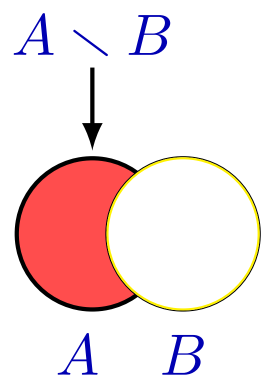

Complement

$A^C = \underbrace{U}_{\text{universe}} \setminus A$

Difference and Complement

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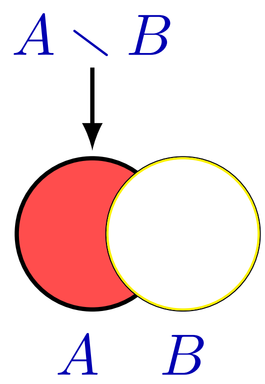

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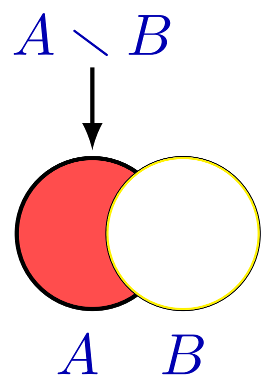

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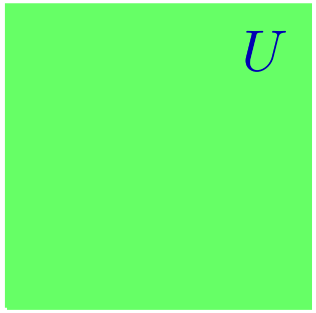

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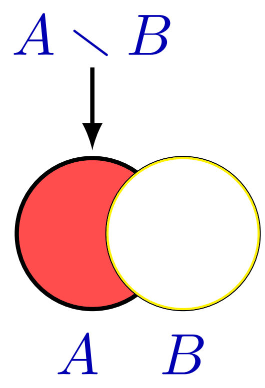

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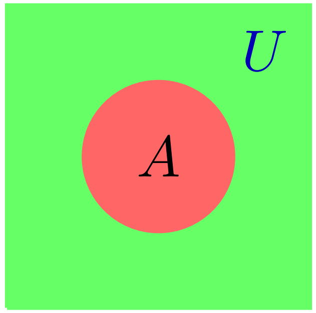

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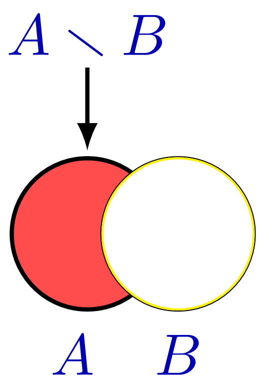

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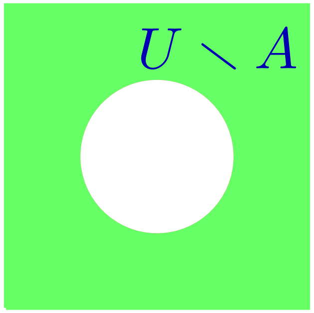

Simplest set-theoretical identities

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Let A be an arbitrary set.

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Definition.

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Definition. Sets A and B are called **disjoint**

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Definition. Sets A and B are called **disjoint** if $A \cap B = \varnothing$.

Set-builder notation

The subset of a set A consisting of the elements x that satisfy a condition $P(x)$ is denoted by $\{x \in A \mid P(x)\}$.

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Vice versa, every subset $B \subset A$ gives rise to a predicate $x \in B$.

Logic vs. set theory

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Proving set-theoretic identities: De Morgan's law

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Example 1.

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Therefore, $(A \cap B)^c \subset A^c \cup B^c \quad (*)$.

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Prove now that $A^c \cup B^c \subset (A \cap B)^c$.

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Example 1. Prove **De Morgan's** law: $(A \cap B)^c = A^c \cup B^c$

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Let $x \in (A \cap B)^c$ Then $x \notin A \cap B \implies \neg(x \in A \wedge x \in B)$

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So $\forall x \in (A \cap B)^c$, we have $x \in A^c \cup B^c$.

Therefore, $(A \cap B)^c \subset A^c \cup B^c \quad (*)$.

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Combining $(*)$ and $(**)$, we get $(A \cap B)^c = A^c \cup B^c$. □

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How to prove set-theoretic identities

Example 2.

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Example 2. Prove that $A \setminus B = A \cap B^c$

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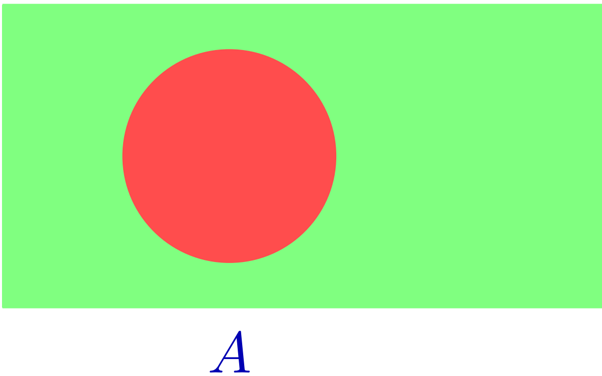

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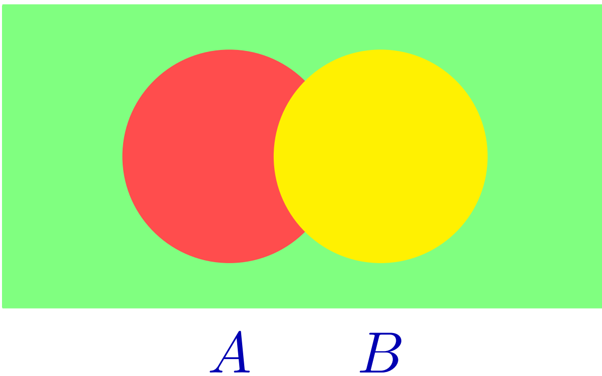

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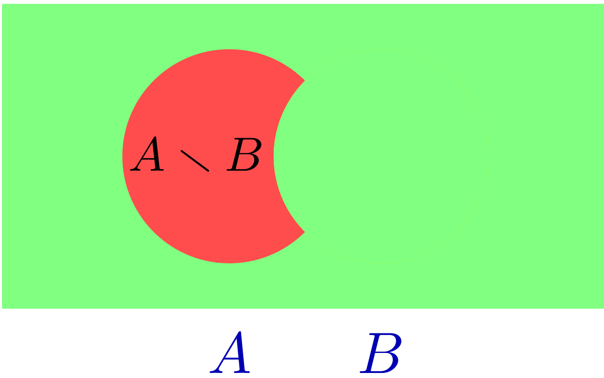

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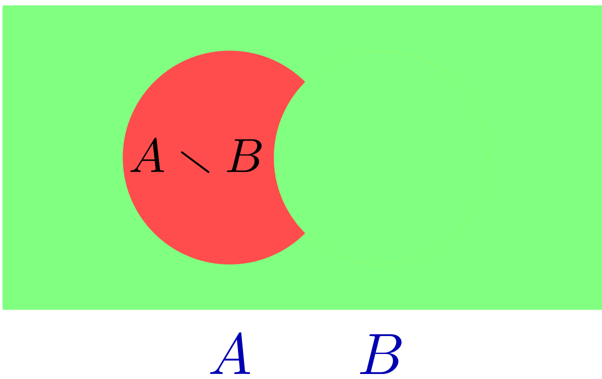

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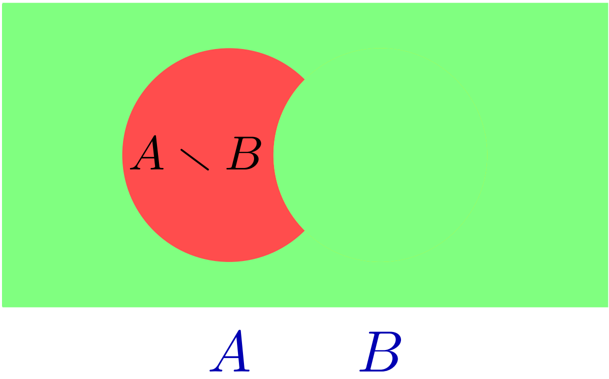

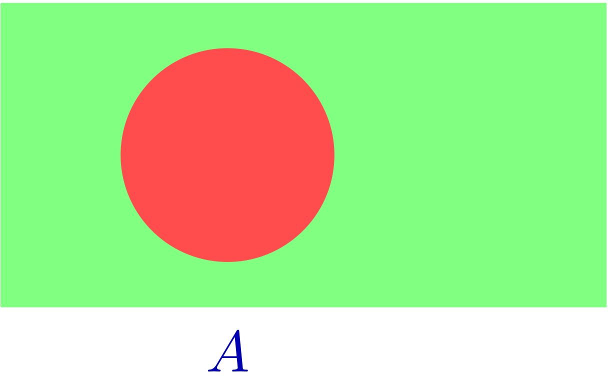

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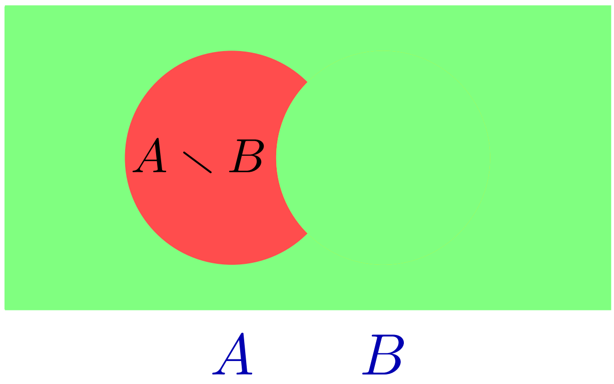

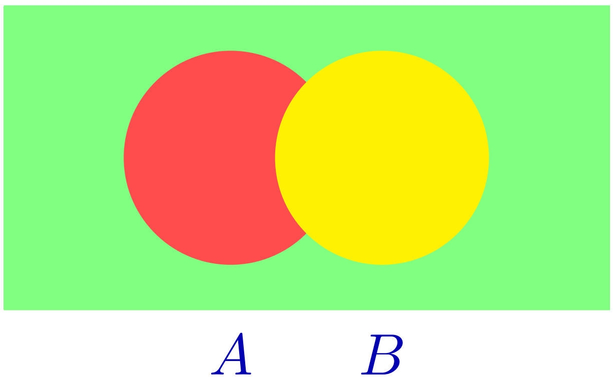

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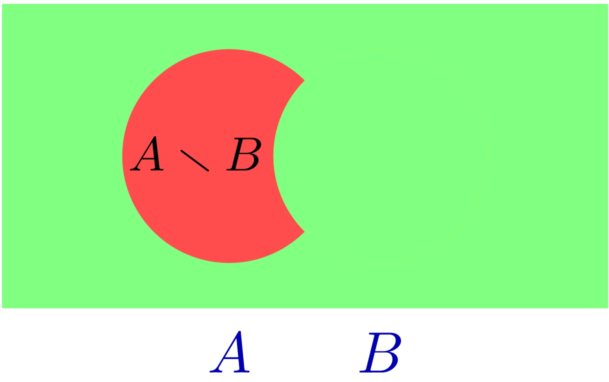

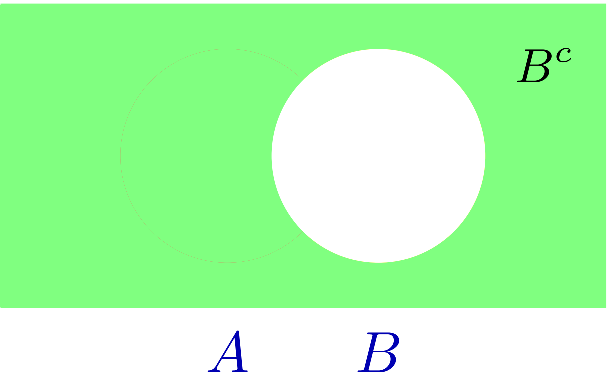

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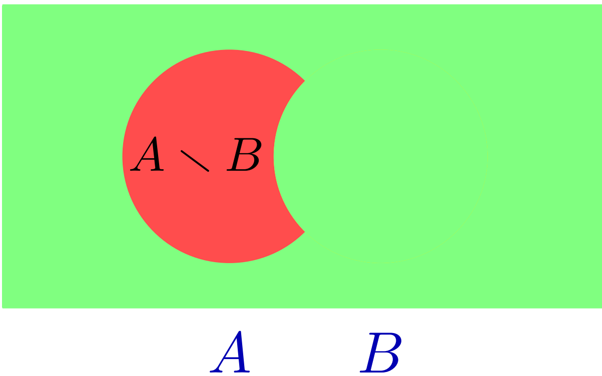

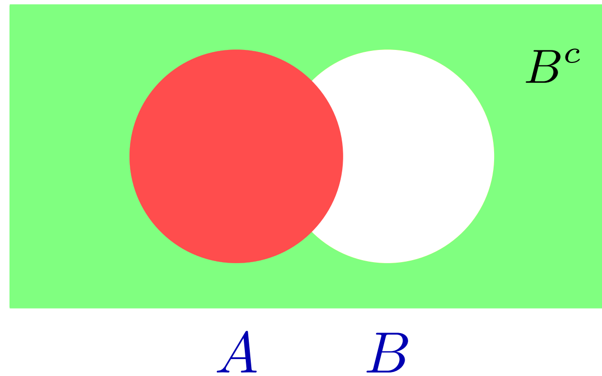

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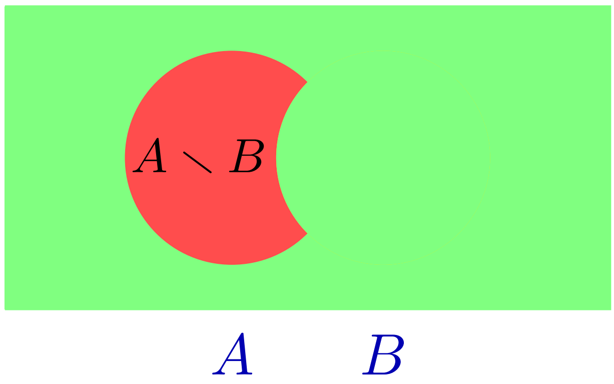

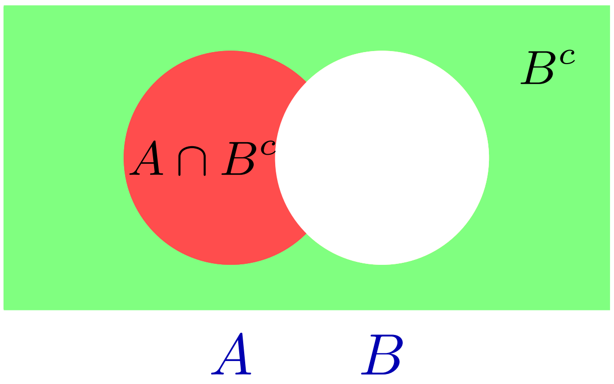

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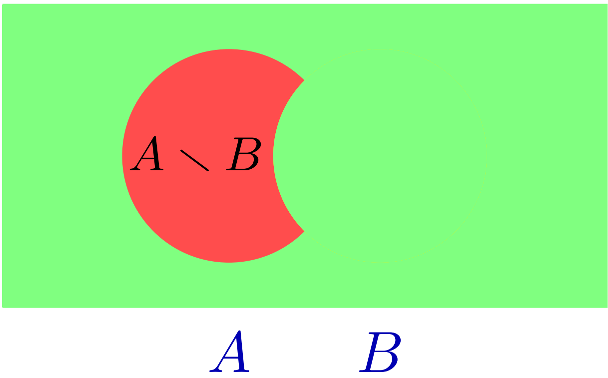

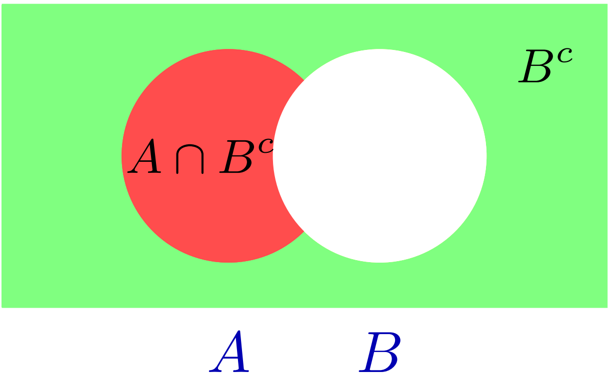

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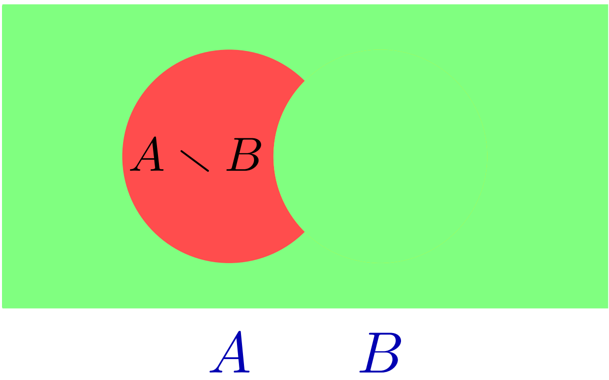

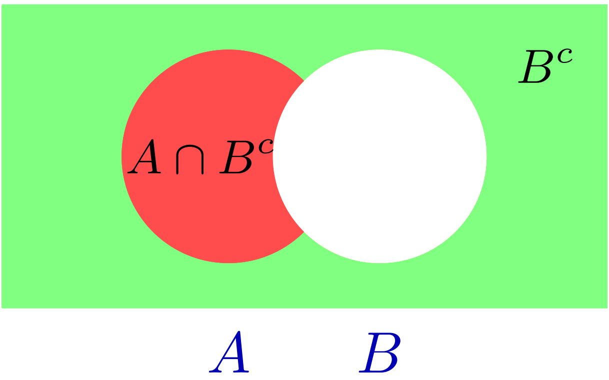

Proof. Alternative 1 (element-wise)

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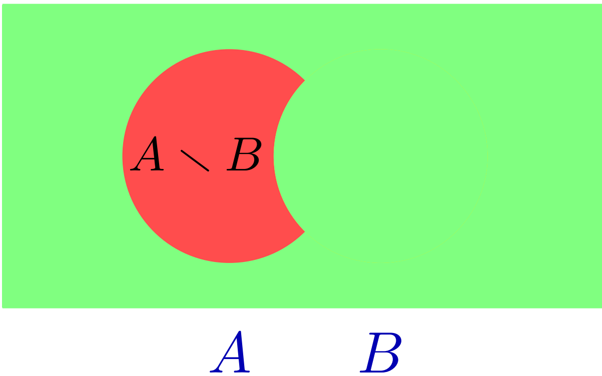

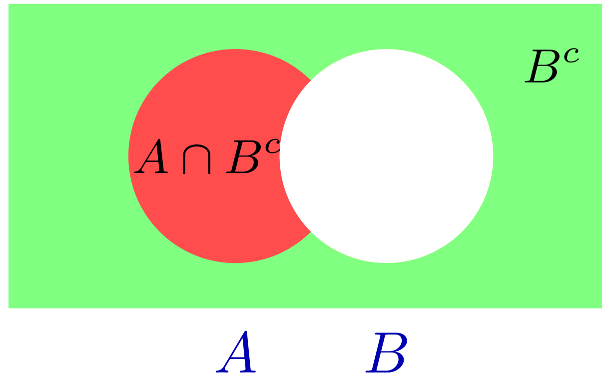

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How to prove set-theoretic identities

Alternative 2

How to prove set-theoretic identities

Alternative 2 (two sets are equal iff each of them is a subset of the other)

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To prove $A \setminus B = A \cap B^c$, we prove that

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Alternative 2 (two sets are equal iff each of them is a subset of the other)

To prove $A \smallsetminus B = A \cap B^c$, we prove that

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Indeed,

$$\left.\begin{array}{l} A \setminus B \subset A \\ A \setminus B \subset U \setminus B = B^c \end{array}\right\} \implies A \setminus B \subset A \cap B^c$$

How to prove set-theoretic identities

Alternative 2 (two sets are equal iff each of them is a subset of the other)

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How to prove set-theoretic identities

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We have got that

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We have got that $A \setminus B \subset A \cap B^c$

How to prove set-theoretic identities

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We have got that $A \setminus B \subset A \cap B^c$ and $A \setminus B \supseteq A \cap B^c$.

How to prove set-theoretic identities

Alternative 2 (two sets are equal iff each of them is a subset of the other)

To prove $A \smallsetminus B = A \cap B^c$, we prove that
$A \smallsetminus B \subset A \cap B^c$ and
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We have got that $A \smallsetminus B \subset A \cap B^c$ and $A \smallsetminus B \supseteq A \cap B^c$.

Therefore, $A \smallsetminus B = A \cap B^c$. $\square$

How to prove set-theoretic identities

Alternative 3

How to prove set-theoretic identities

Alternative 3 (by truth table)

How to prove set-theoretic identities

Alternative 3 (by truth table)

	$x \in A$	$x \in B$	$x \notin B$	$\underbrace{x \in A \wedge x \notin B}_{x \in A \setminus B}$	$\underbrace{x \in A \wedge x \in B^c}_{x \in A \cap B^c}$
1	T	T	F	F	F
2	T	F	T	T	T
3	F	T	F	F	F
4	F	F	T	F	F

How to prove set-theoretic identities

Alternative 3 (by truth table)

	$x \in A$	$x \in B$	$x \notin B$	$\underbrace{x \in A \land x \notin B}_{x \in A \setminus B}$	$\underbrace{x \in A \land x \in B^c}_{x \in A \cap B^c}$
1	T	T	F	F	F
2	T	F	T	T	T
3	F	T	F	F	F
4	F	F	T	F	F

Since the last two columns of the truth table are identical,

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1	T	T	F	F	F
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Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$.

How to prove set-theoretic identities

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	$x \in A$	$x \in B$	$x \notin B$	$\underbrace{x \in A \land x \notin B}_{x \in A \setminus B}$	$\underbrace{x \in A \land x \in B^c}_{x \in A \cap B^c}$
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Remark.

How to prove set-theoretic identities

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Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$.

Remark. The universe can be presented as a **disjoint union**

How to prove set-theoretic identities

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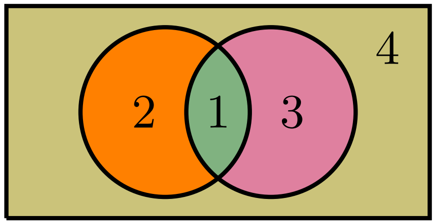
Since the last two columns of the truth table are identical, $A \setminus B = A \cap B^c$.

Remark. The universe can be presented as a **disjoint union**

$$U = (A \cap B) \cup (A \setminus B) \cup (B \setminus A) \cup (A \cup B)^c$$



Alternative 3 (by truth table)

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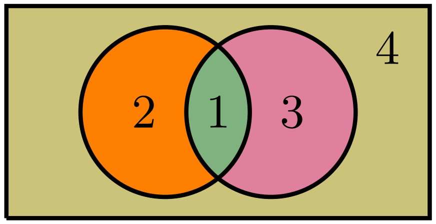

What does this formula remind you?

Alternative 3 (by truth table)

	$x \in A$	$x \in B$	$x \notin B$	$\underbrace{x \in A \wedge x \notin B}_{x \in A \setminus B}$	$\underbrace{x \in A \wedge x \in B^c}_{x \in A \cap B^c}$
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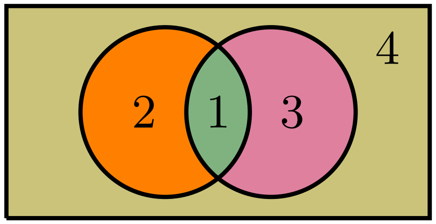

What does this formula remind you? Is it related to disjunctive normal form?

How to prove set-theoretic identities

How to prove set-theoretic identities

Example 3.

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

Proof.

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \smallsetminus B = \varnothing$ for any sets A, B.

Proof. Let $A \subset B$.

Example 3. Prove that $A \subset B \iff A \smallsetminus B = \varnothing$ for any sets A, B.

Proof. Let $A \subset B$. Then

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

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How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

Proof. Let $A \subset B$. Then

$\forall x \in A \quad x \in A \implies x \in B$

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

Proof. Let $A \subset B$. Then
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How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

Proof. Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
So any x in A doesn't belong to B^c.

How to prove set-theoretic identities

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Proof. Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
So any x in A doesn't belong to B^c. Therefore,

How to prove set-theoretic identities

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Proof. Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
So any x in A doesn't belong to B^c. Therefore, $A \cap B^c = \varnothing$.

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$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
So any x in A doesn't belong to B^c. Therefore, $A \cap B^c = \varnothing$.

But $A \cap B^c = A \setminus B$,

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But $A \cap B^c = A \setminus B$,

hence $A \setminus B = \varnothing$
We have proven that $A \subset B \implies A \setminus B = \varnothing \quad (*)$

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Prove now the opposite implication.

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

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$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
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Prove now the opposite implication.

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How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

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But $A \cap B^c = A \setminus B$,

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We have proven that $A \subset B \implies A \setminus B = \varnothing \quad (*)$

Prove now the opposite implication.

Let $A \setminus B = \varnothing$. Then $A \cap B^c = \varnothing$. Therefore,

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Therefore,

How to prove set-theoretic identities

Example 3. Prove that $A \subset B \iff A \setminus B = \varnothing$ for any sets A, B.

Proof. Let $A \subset B$. Then
$\forall x \in A \quad x \in A \implies x \in B \implies x \notin B^c$.
So any x in A doesn't belong to B^c. Therefore, $A \cap B^c = \varnothing$.

But $A \cap B^c = A \setminus B$,

hence $A \setminus B = \varnothing$
We have proven that $A \subset B \implies A \setminus B = \varnothing \quad (*)$

Prove now the opposite implication.

Let $A \setminus B = \varnothing$. Then $A \cap B^c = \varnothing$. Therefore,

$\forall x \in A \quad x \in A \implies x \notin B^c \implies x \in B$. By this, $A \subset B$.

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