Problem 1.

A professor discusses results of the exam with the students. Rewrite symbolically each of the professor's statements using the following notations: X is the set of all students, Y is the set of all problems given to the students, and $P(x, y)$ is the predicate "Student x solved problem y on the exam."

	Professor's statement	Symbolic statement
1	Everyone solved all problems.	
2	Nobody solved all problems.	
3	Nobody solved any problem.	
4	Everyone solved at least one problem	
5	Some problem was solved by everybody.	
6	All the problems were solved by someone.	
7	Someone did not solve exactly one problem.	
8	Some problem was not solved by anyone.	
9	Someone solved all the problems.	
10	At least someone solved at least one problem.	

Which of the statements 1 – 9 are equivalent?

Problem 2. Ann studied logic only if Ben did, but it's not true that if Den studied logic, then Ben did. Who did study logic? Provide a complete justification of your answer.

Problem 3. Let P, Q and R be propositions. Consider the following propositional form:

$$P \implies Q \vee R \iff P \wedge \neg R \implies Q$$

Indicate the order of logic operations. Is the propositional form tautology, contradiction or neither tautology nor contradiction?

Problem 4. Let x be a real number. Consider the statement

$$x^2 = 4 \quad \text{only if} \quad x = 2 \ \text{ or } \quad x < 0$$

(a) Rewrite the statement in symbolic form using logic connectives. Indicate the order of logic operations.

(b) Rewrite the statement in a logically equivalent form using the word "whenever".

(c) Construct a useful denial (in affirmative terms) of the statement.

(d) Find all values of x for which the statement is true. Justify your answer!

Problem 5.

(a) Formulate a sufficient but not necessary condition for an integer to be divisible by 6.

(b) Formulate a necessary but not sufficient condition for an integer to be divisible by 6.

(c) Formulate a sufficient and necessary condition for an integer to be divisible by 6.

Problem 6. Consider the following statements:

(1) A quadrilateral has four congruent sides.

(2) A quadrilateral is a square.

(3) A quadrilateral is a rectangle.

(4) A quadrilateral has two right angles.

Insert into the blanks the appropriate logical connectives ($\Longrightarrow$, $\Longleftarrow$, $\Longleftrightarrow$):

(1) __________ **(2)** __________ **(3)** __________ **(4)**

Insert into the blanks the words "sufficient but not necessary", "necessary but not sufficient", "sufficient and necessary". Provide explanations.

(1) is ______________________________ for **(2)**

(2) is ______________________________ for **(1)**

(2) is ______________________________ for **(3)**

(3) is ______________________________ for **(2)**

(3) is ______________________________ for **(4)**

(4) is ______________________________ for **(3)**

Problem 7. Recall a theorem from Calculus:

If a series $\sum_{n=1}^{\infty} a_n$ *converges, then* $\lim_{n\to\infty} a_n = 0$.

(a) Explain why this theorem is called a **necessary** condition for convergence. (You do not need to prove the theorem.)

(b) Formulate the contrapositive of the theorem. Is the contrapositive true? Provide explanations.

(c) Formulate the converse of the theorem. Is the converse true? Provide explanations.

(d) Formulate the inverse of the theorem. Is the inverse true? Provide explanations.

Problem 8. Consider the following statement:

"For any non-zero real numbers a and any real number b, the equation $ax = b$ has a unique real solution."

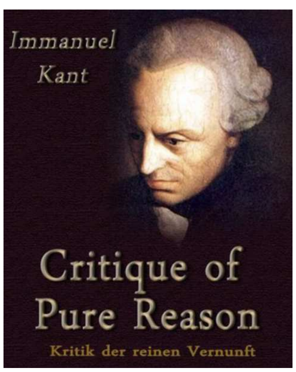
Write down this statement using symbols only (with no words).

Problem 9. Let $P(x, y)$ be the predicate $x + y > 1 \implies y \le |x|$.

(a) Show on the coordinate plane **all** points (x, y) for which $P(x, y)$ holds true.

(b) Show on the coordinate plane **all** points (x, y) for which $P(x, y)$ holds false.

(c) Construct all possible quantified sentences using the predicate $P(x, y)$ and the universal and existential quantifiers. Determine the truth value of each sentence. Justify your answers.



Problem 10.

All our knowledge beginſ with experience,
but not all of it ariſeſ from experience.

(a) Write down the statement above in symbolic form.
You have to introduce appropriate notations and use quantifiers.

(b) Write down an equivalent statement which doesn't contain negation of a quantified sentence.