Practice Midterm 2

Bézou Theorem

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Division with remainder **Remainder theorem**

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What are its periods?

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(d) $\bigcap_{i=1}^{\infty} A_i = \varnothing$.

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and the quotient set.

Modular arithmetic?

Find all integer solutions (x, y) for equation $xy + 2x + 3y = 7$.

Algebraic numbers

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How?