Last digit

Last digit

Problem 3.

Last digit

Problem 3. Find the last digit of 2022^{2023}.

Last digit

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Solution.

Last digit

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Solution. $2022^{2023} \equiv ? \mod 10$

Last digit

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$$2022 \equiv 2 \mod 10$$

Last digit

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Solution. $2022^{2023} \equiv ? \mod 10$

$$2022 \equiv 2 \mod 10 \implies 2022^{2023} \equiv 2^{2023} \mod 10$$

Last digit

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$$2022 \equiv 2 \mod 10 \implies 2022^{2023} \equiv 2^{2023} \mod 10$$

$$2^{2023} \equiv ? \mod 10$$

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Problem 3. Find the last digit of 2022^{2023}.

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$$2022 \equiv 2 \mod 10 \implies 2022^{2023} \equiv 2^{2023} \mod 10$$

$$2^{2023} \equiv ? \mod 10$$

$2^1 \equiv 2 \mod 10$

Last digit

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$$2022 \equiv 2 \mod 10 \implies 2022^{2023} \equiv 2^{2023} \mod 10$$

$$2^{2023} \equiv ? \mod 10$$

$2^1 \equiv 2 \mod 10$

$2^2 \equiv 4 \mod 10$

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$$2^{2023} \equiv ? \mod 10$$

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$2^2 \equiv 4 \mod 10$

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. . . . . .

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$$2022 \equiv 2 \mod 10 \implies 2022^{2023} \equiv 2^{2023} \mod 10$$

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$$\left.\begin{aligned} 2^1 &\equiv 2 \mod 10 \\ 2^2 &\equiv 4 \mod 10 \\ 2^3 &\equiv 8 \mod 10 \\ 2^4 &\equiv 6 \mod 10 \\ 2^5 &\equiv 2 \mod 10 \\ 2^6 &\equiv 4 \mod 10 \\ &\cdots\cdots\cdots\cdots \end{aligned}\right]$$

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. . . . . .

cycle of length 4

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.....

cycle of length 4

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.....

cycle of length 4

$2023 \equiv ? \mod 4$

$2023 \equiv 3 \mod 4$

Last digit

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.....

cycle of length 4

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.....

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.....

cycle of length 4

$2023 \equiv ? \mod 4$

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$2^{2023} \equiv 8 \mod 10$

$2022^{2023} \equiv 8 \mod 10$

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cycle of length 4

$2023 \equiv ? \mod 4$

$2023 \equiv 3 \mod 4$

$2^{2023} \equiv 8 \mod 10$

$2022^{2023} \equiv 8 \mod 10$

Answer:

the last digit of 2022^{2023} is 8.

Diophantine equations

Diophantine equations

Problem 3.

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Problem 3. Prove that the equation $x^2 - 3y^2 = 17$

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$$[x^2 - 3y^2]_3 = [17]_3$$

Diophantine equations

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$$[x^2]_3 - [3y^2]_3 = 2$$

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$x \mod 3$	$x^2 \mod 3$
0	0
1	1
-1	1

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So $x^2 \equiv 0$ or $1 \mod 3$,

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So $x^2 \equiv 0$ or $1 \mod 3$, and $x^2 \not\equiv 2 \mod 3$.

Diophantine equations

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$x \mod 3$	$x^2 \mod 3$
0	0
1	1
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So $x^2 \equiv 0$ or $1 \mod 3$, and $x^2 \not\equiv 2 \mod 3$.

Therefore, the original equation has **no** integer solutions.

Reduction modulo m

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As we know, an equivalence relation on a set X

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This map has many names:

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Ring homomorphism

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Definition. Let R, S be rings.

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Remark. $f(a+b) = f(a) + f(b)$.

$\uparrow$ addition in R $\quad$ $\uparrow$ addition in S

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↑ addition in R ↑ addition in S

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↑ multiplication in R ↑ multiplication in S

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$$f(ab) = f(a)f(b)$$

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Theorem.

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Therefore, f is a ring homomorphism. $\square$

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My mathematical ancestors

Mathematics Genealogy Project https://www.genealogy.math.ndsu.nodak.edu/

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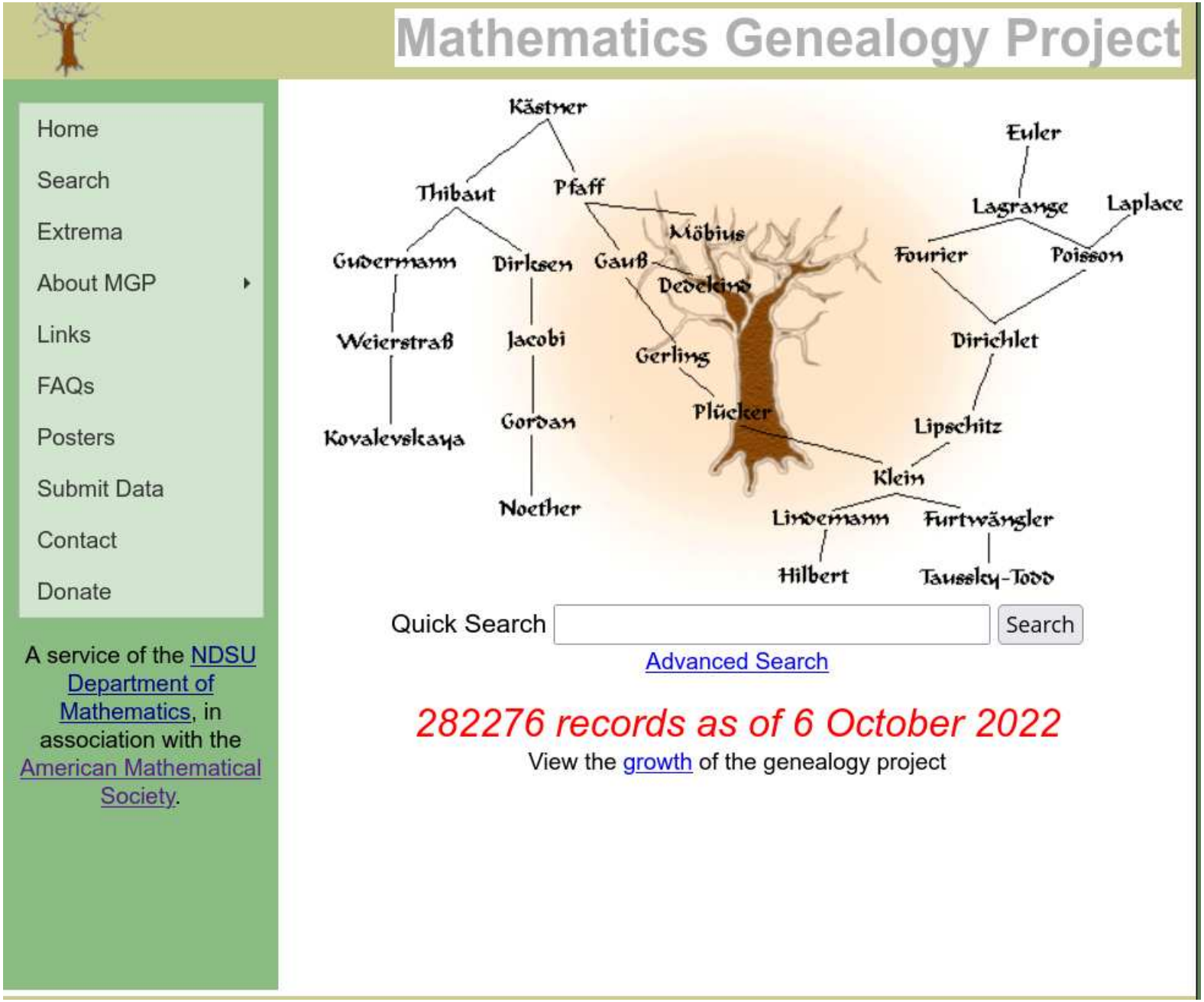

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