MAT 250 **Homework 1**

Problem 1. Let a, b be integers which are both not equal to zero. An integer d is called the *greatest common divisor* of a and b if d is the greatest positive integer dividing both a and b.

Notation: $d = \gcd(a, b)$.

1.1 Give a symbolic form of this definition (don't use words). Which variables are free and which are bound in this definition?

1.2 Construct a useful denial, that is explain what it means that $d \neq \gcd(a, b)$. Give the denial in two forms: in words and in symbols only.

Problem 2. Let X be a subset of the set of real numbers. The *supremum* (or the *least upper bound*) of X is the least real number that is greater than or equal to every element of X. Notation: $\sup X$.

2.1 Give a symbolic form of this definition. Formulate what it means that a number is not a supremum of X, both in words and symbols.

2.2 Let $X = (0, 1)$. What is $\sup X$? Justify your answer using the definition.

2.3 Let $\mathbb{N} = \{1, 2, 3, \dots\}$ be the set of natural numbers. What is $\sup \mathbb{N}$? Justify your answer.

Problem 3. 3.1 Show that the set of all numbers of the form $a + b\sqrt{2}$ where $a, b \in \mathbb{Z}$, is a ring with respect to usual addition and multiplication. Is this ring commutative? Is this a ring with unity? Justify your answers!

3.2 Is the set of all numbers of the form $a + b\sqrt[3]{2}$ where $a, b \in \mathbb{Z}$, a ring? Explain!

Problem 4. Prove that if a sum of n real numbers is equal to 1, then at least one of the numbers is less than or equal to 1.