NAME **Homework 5** MAT 364 October 4, 2019

1. Prove that the intersection of two closed sets is closed (you have to write down a proof using the definition of closed set.)
2. Give a topology on a set X, for which there exists a non-empty subset A of X, such that $A \neq X$ and A is both open and closed.
3. Prove that the union of two closed sets is closed. (You can assume your sets are in $\mathbb{R}^n$ but it is not necessary).
4. Consider $X = \mathbb{R}$. What are the limit points of the following subsets
   a $\mathbb{Q}$, the set of rational numbers.
   b $[1, 2] \cap \{3\}$
   c $\mathbb{R} \setminus \mathbb{Z}$.
   d The graph of the function f from the set of positive real numbers to the $\mathbb{R}$, defined by $f(x) = \sin(1/x)$.
5. Give an examples of two subsets of $\mathbb{R}^2$, X, Y, such that $X \subset Y$, B is open in A, and A is not open in R^2.
6. Let $X = [1, 2] \cap \{3\}$. Determine whether the following subsets X are open on closed in X.
   a $\{3\}$
   b $\{1\}$
   c $[1, 1/2)$.
7. Consider $X = \{(x, y) \in \mathbb{R}^2 : x \leq 1 - y, -1 \leq x \leq 1, \text{ and } x \geq y - 1\}$ and the relation $\sim$ on X given by
   a (P, P) for all P in X.
   b $(x, 1 - x) \sim (x, 0)$ for all $x \in [0, 1]$
   c $(x, 0) \sim (x, 1 - x)$ for all $x \in [0, 1]$
   d $(x, 1 + x) \sim (x, 0)$ for all $x \in [-1, 0]$
   e $(x, 0) \sim (x, 1 + x)$ for all $x \in [-1, 0]$

   Prove that $\sim$ is an equivalence relation and find $X/\sim$.
8. Consider the set $X = \{1, 2, 3, 4\}$ and the list of subsets of X, $L = \{\{1, 2\}, \{2, 3\}\}$.
   a Explain why L is not a topology on X.
   b Add the smallest possible number of subsets to L to obtain a topology.

c Determine whether the topology you obtained in b is Hausdorff.

9. Let $X = \{1, 2, 3\}$, and $T = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 3\}\}$.

   a Prove that T is a topology on X.

   b Find a function from X to $\mathbb{R}$ that is not continuous.

10. Problems 1.2: 22 an 24.