NAME **Homework 4** MAT 364 September 20, 2019

For problemsn 1, 2 and 3, a set X and a surjective function f from X to a set Y is given. This defines an equivalence relation R on X, in the following way. A pair $(x, y) \in X \times X$ is in the relation R if $f(x) = f(y)$. You have to describe the quotient topology on the set $X/\equiv$.

Problem 1

$$X = [0,1],\ f(x) = \begin{cases} x & \text{if } x \notin \{0, 1/2, 1\} \\ 0 & \text{otherwise.} \end{cases}$$

Problem 2

$$X = [0,1] \times [0,1] \text{ (a unit square)},\ f(x,y) = \begin{cases} (x,y) & \text{if } x \neq 0 \\ (1, 1-y) & \text{if x=0} \end{cases}$$

Problem 3

$X = \{(x,y)/x^2 + y^2 \leq 1\}$ (the closed unit ball). The map f goes from X to the interval $[0,1]$, $f(x,y) = x^2 + y^2$.

Problem 4

$X = \{(x,y)/x^2 + y^2 \leq 1\}$ (the closed unit ball). The equivalence relation contains all pairs of points $((x,y),(x',y'))$ such that one of the following holds

1. $x = x'$ and $y = y'$
2. $x^2 + y^2 = x'^2 + y'^2 = 1$

either $x = x'$ and y

Problem 5

Problem 28 of Section 1.1 of the textbook.

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Problem 6

Let $X = \{a, b, c, d\}$. Determine which of the lists of subsets of X given in Figure 1 is a topology. Justify your answer. In each case, we assume that the list contains the empty set.

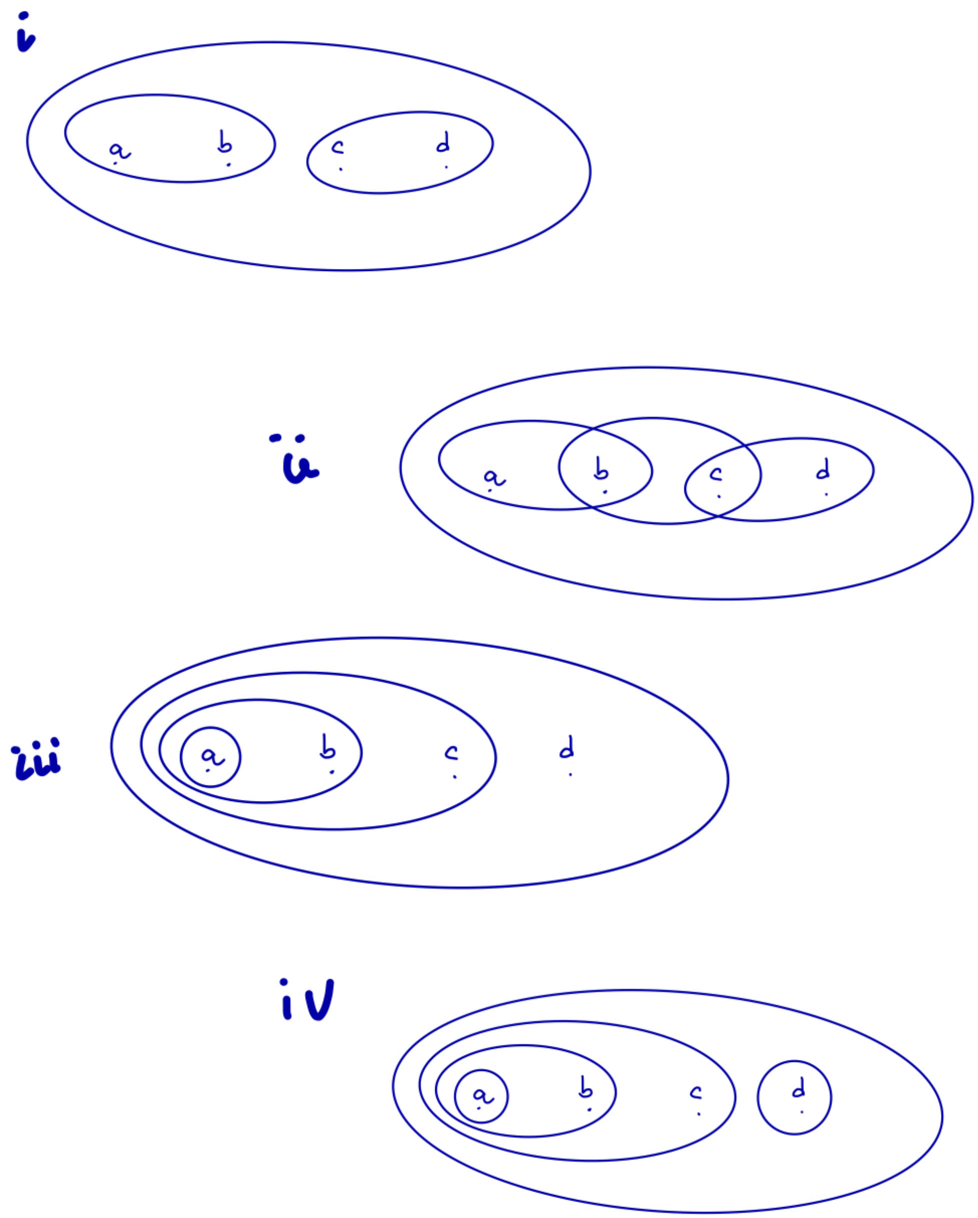

Figure 1: Which are topologies?