NAME

Homework 4

MAT 364
September 25, 2019

Problem 1

Let $X = \{a, b, c, d\}$. Determine which of the lists of subsets of X given in Figure 1 is a topology. Justify your answer. In each case, we assume that the list contains the empty set.

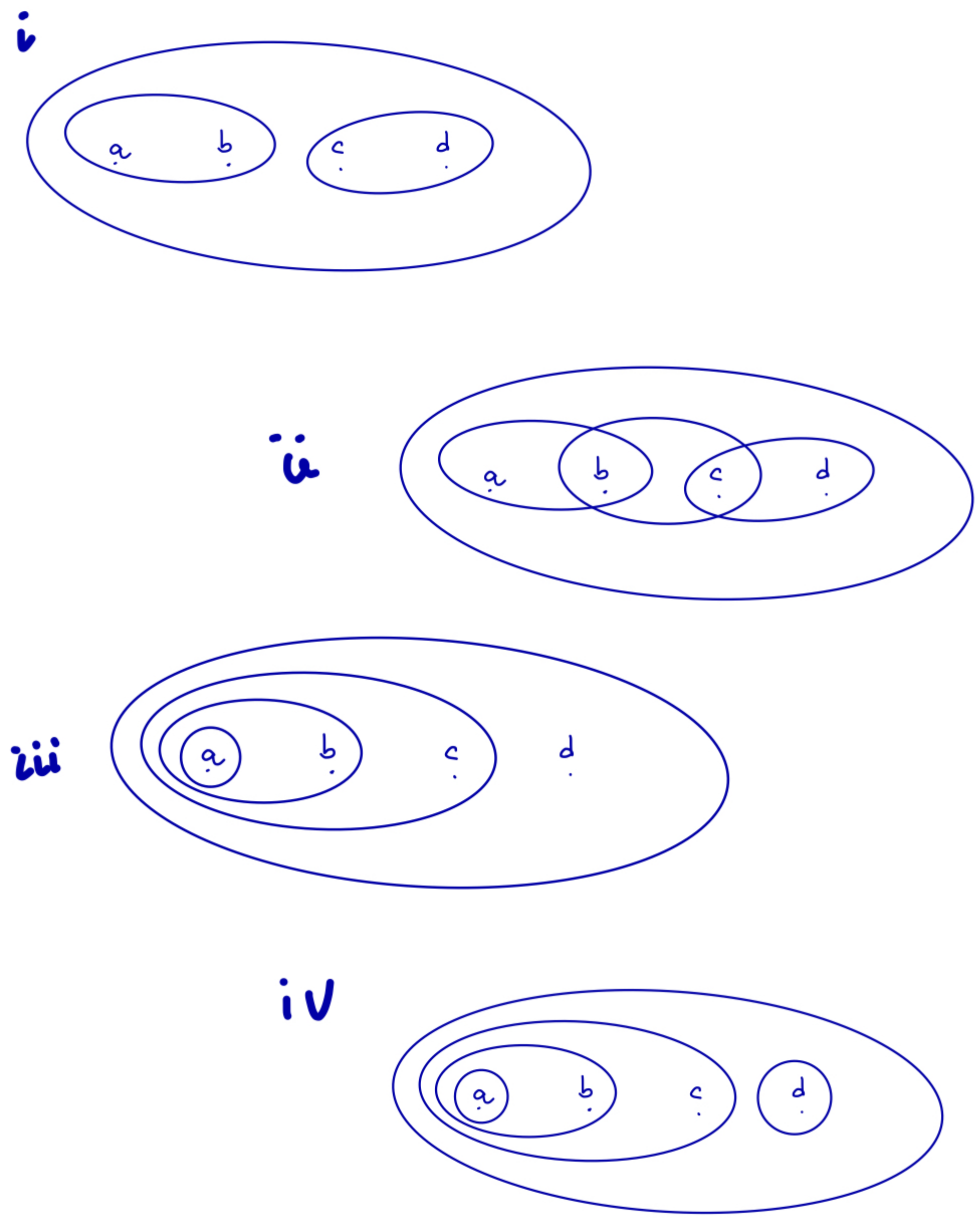

Figure 1: Which are topologies?

Problem 2

Problem 28 of Section 1.1 of the textbook.

Problem 3

Let $X = [0, 1]$. Find an equivalence relation on X such that $X/\sim$ is the letter Y.

In problems 4 to 7 an equivalence relation R on a set X is given. Convince yourself that each relation is indeed an equivalence relation (you are not required to write down a proof). Describe the open sets of $X/\sim$ and, if possible draw $X/\sim$.

Problem 4

$X = [0, 1]$ and the relation R contains all pairs of the form (x, x'), such that either

1. $x = x'$.
2. $\{x, x'\} \subset \{0, 1/2, 1\}$.

Problem 5

$X = \{(x, y)/x^2 + y^2 \leq 1\}$ (the closed unit ball). A map f goes from X to the interval $[0, 1]$ is defined by $f(x, y) = x^2 + y^2$. The map f defines an equivalence relation R on X, in the following way. A pair $(x, y) \in X \times X$ is in the relation R if $f(x) = f(y)$.

Problem 6

$X/\sim$ where $X = [0, 1] \times [0, 1]$ (a unit square) and the equivalence relation R contains all pairs of points $((x, y), (x', y'))$ such that one of the following holds

1. $x = x'$ and $y = y'$,
2. $((0, y^2), (1, y))$,
3. $((1, y), (0, y^2))$.

Problem 7

$X = \{(x,y)/x^2+y^2 \leq 1\}$ (the closed unit ball) and the equivalence relation R contains all pairs of points $((x,y),(x',y'))$ such that one of the following holds

1. $x = x'$ and $y = y'$
2. $x^2+y^2 = x'^2+y'^2 = 1$