NAME: SB ID:

MAT 364 Topology and Geometry - Nov 15, Midterm Ii, Fall 2018

Problem	1	2	3	4	5	6-EC	Total
Score 10	10	10	10	10	10		50
Total Score							

(i) Full credit credit is given for providing a proof with appropriate justification (appropriate means providing only the relevant and necessary steps to obtain the proof).

(ii) Write your answers in complete (and correct!) English sentences.

(iii) Cross out the work you do not want to be graded.

(1) Prove that if X and Y are topological spaces and $X \times Y$ is compact then X and Y are compact. (Hint: Proving that certain function is continuous gives you a quick proof.)

(2) Give a counterexample to the following statement: If X and Y are topological spaces, X is Hausdorff and there exist a continuous surjective map from X to Y then Y is Hausdorff. (Observe that this implies the quotient of a Hausdorff space may not be Hausdorff).

(3) Give a representation of the surface $K\#P$ connected sum of a Klein bottle and a projective plane as hexagon with pairs of edges glued together.

(4) Which of the surfaces in the list, sphere, connected sum of n tori, connected sum n projective planes, is homeomorphic to $T\#P$ (the connected sum of a torus and a projective plane) by the Classification of Surfaces Theorem:. If $T\#P$ is not homeomorphic to a sphere, also determine n.

(5) The following list of sets $\{\ldots,(2n-1,2n),[2n,2n+1],(2n+1,2n+2),[2n+2,2n+3]\ldots\}$ is a partition of the real line $\mathbb{R}$. Determine the corresponding quotient topology.

(6) (Extra credit) Consider the family of functions $f_r : (-1,1) \longrightarrow \mathbb{R}$, $f_r(x) = r + \frac{1}{1-x^2}$. This family determines the partition of $[-1,1] \times \mathbb{R}$, $\{J_1, J_{-1},\} \cup \{I_r\}_{r\in\mathbb{R}}$ where $J_1 = \{1\} \times \mathbb{R}$, $J_0 = \{0\} \times \mathbb{R}$, and $I_r = \{(x, f_r(x)), x \in \mathbb{R}\}$.

Determine the quotient topology on the set of elements of the partition, $\{J_1, J_{-1},\} \cup \{I_r\}_{r\in\mathbb{R}}$.