MAT 568

Topics: Hadamard's Theorem, Spaces of Constant Curvature

Reading

- do Carmo, Chapter 7, 8.1–8.4

Exercises (to do on your own)

(1) Find a local diffeomorphism $f : M \to N$ that is not a covering map. Also, put a complete Riemannian metric g on N such that f^*g is not complete on M.

(2) Prove that every manifold M admits a complete Riemannian metric. One way to do this is suggested as follows. Without loss of generality, say M is connected, and start with any metric. Let U_n be an infinite sequence of open sets whose union is M and such that U_n is compact and contained in U_{n+1}. Try to make the metric "big" on $U_{n+1} \setminus U_n$, and invoke exercise 5 of Chapter 7 of do Carmo. Why does such a sequence $\{U_n\}$ exist?

(3) Let M be a complete, simply-connected Riemannian manifold containing a pole p. This means p has no conjugate points. What can you conclude about the topology of M? Find a surface of strictly positive curvature that contains a pole.

Problems (to turn in)

(1)
  (a) do Carmo, Chapter 8, exercise 4, pg. 181 (Lens spaces, closed geodesics)
  (b) Find a non-cyclic finite group acting on S^3 properly discontinuously by isometries.
  (c) s Find a group action on S^3 by isometries that is not properly discontinuous.

(2) do Carmo, Chapter 8, exercise 9, pg. 186 (connection of Riemannian submersion). Read exercise 8 for the definition.

(3) do Carmo, Chapter 8, exercise 10, pg. 186 (curvature of Riemannian submersion).

(4) do Carmo, Chapter 8, exercise 14, pg. 190 (locally symmetric spaces and reversing geodesics)