MAT 568

Topics: Bonnet–Myers, Synge–Weinstein, Rauch, Morse Index Theorem

Reading

- do Carmo, Chapter 9, 10.1–10.2, 11

Exercises (to do on your own)

(1) do Carmo, Chapter 9, exercise 2, pg. 207. Note that a paraboloid satisfies the hypotheses, yet has positive Gauss curvature everywhere.
(2) do Carmo, Chapter 10, exercise 2, pg. 237. (Klingenberg?s Lemma implies Hadamard?s theorem.)
(3) What can you say about the fundamental group of a connected Einstein manifold of positive scalar curvature?
(4) Prove that among compact Riemannian manifolds, it is not possible to find a universal bound on diameter in terms of volume, or vice versa.
(5) Consider $S^2 \times S^2$ with the product metric, which has $K \geq 0$. Let f be the map that is the antipodal map on each factor, which preserves the standard orientation on $S^2 \times S^2$. Certainly, f has no fixed points. What exactly goes wrong in the proof of Weinstein?s theorem?

Problems (to turn in)

(1) do Carmo, Chapter 9, exercise 3, pg. 207. (generalization of Bonnet–Myers).
(2) do Carmo, Chapter 10, exercise 3, pg. 237. Also: give a counterexample if you merely assume $\mathrm{Ric} \leq 0$. (Hint: use the last problem of the midterm, with $\psi(t) = \cosh(t)$, and note that the sphere at $t = 0$ is round and totally geodesic.)
(3) do Carmo, Chapter 11, exercise 6, pg. 252. (Hint: use previous exercises in do Carmo.) A major generalization of this is the Cheeger–Gromoll splitting theorem, which says: if a complete manifold M of nonnegative Ricci curvature contains a line, then M must be isometric to $N \times \mathbb{R}$ for some N.