

HOMEWORK 2

- (1) What is the homology of the space $(\mathbb{R}^2 \times S^n)/\mathbb{Z}^2$ where $(0, 1) \in \mathbb{Z}^2$ sends $((x, y), z)$ to $((x, y + 1), z)$ and $(1, 0)$ sends $((x, y), z)$ to $((x + 1, y), -z)$ and where $n > 0$?
- (2) Consider a fibration over a closed manifold whose fiber has finite total Betti number. Show that it has non-zero reduced homology.
- (3) Let B be a simply connected four dimensional CW complex satisfying $H^3(X; \mathbb{Z}) = 0$ and let $E \rightarrow B$ be a fibration with fiber $\mathbb{CP}^2$. Does it admit a section?
- (4) Show that if a complex surface has trivial first Chern class and trivial Euler class then its tangent bundle is trivial as a complex vector bundle.