MAE 301/501 HOMEWORK–6 DUE ON THURSDAY, NOVEMBER 4

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

This document has two pages.

(1) (a) Write the power series definition of the exponential function.

(b) Use this definition to prove that the exponential function satisfies the relation $e^{x+y} = e^x e^y$, for all real numbers x and y.

(2) The exponential function is also defined as the following limit:

$$e^x := \lim_{n\to\infty}(1+\frac{x}{n})^n.$$

(a) Briefly explain how this definition is related to continuously compounded interest.

(b) Use this definition to prove that the function satisfies the relation $e^{x+y} = e^x e^y$, for all real numbers x and y.

(3) We also showed that the exponential function arises as the unique solution to the initial value problem:

$$f'(x) = f(x)$$
$$f(0) = 1.$$

Using the uniqueness of solutions to the initial value problem, prove that the exponential function, defined as this unique solution, satisfies the relation $e^{x+y} = e^x e^y$.

(4) Using the definition of the logarithmic function, as well as any results we already proved in class, to prove the following:

(a) Prove, for all $x > 0$, that $\log_a \frac{1}{x} = -\log_a x$.

(b) Prove, for all $x > 0$, that $\log_a x^y = y \log_a x$.

(c) Prove that $\frac{d}{dx}\ln x = \frac{1}{x}$.

(d) Prove that $\frac{d}{dx}\log_a x = \frac{1}{x(\ln a)}$.

(e) Prove that $\ln x < x$ for all $x > 0$.

(f) Find $\log_2 4$.

(g) Find $2^{\log_2 4}$.

(h) Solve for x: $\log_x 16 = 2$.

(i) Show that, for all x, $\frac{\log_2 x}{\log_4 x} = 2$.

(j) Solve for x: $2^{2\log 2(6+x)+2} = 16$.

(k) Prove that, for all positive a and b, $(\log_a b)(\log_b a) = 1$.