

MAE 301/501 HOMEWORK-3 DUE ON THURSDAY, SEPTEMBER 30

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

- (1) The graph of the function

$$y = \frac{1}{x}$$

looks like the graph of a hyperbola. Prove that this is the case. One way is to prove that it can be rotated 45° onto the graph of an equation that has the general form for a parabola. Another way might be to prove that the set of points defined by $y = \frac{1}{x}$ satisfies the definition of a hyperbola.

- (2) The line $y = mx + b$ is a slant asymptote of a function f if

$$\lim_{x \rightarrow \infty} |f(x) - (mx + b)| = 0$$

or if

$$\lim_{x \rightarrow -\infty} |f(x) - (mx + b)| = 0.$$

Determine whether or not the graph of a *rational function* can cross its slant asymptote, and prove your result.

- (3) Can the graph of a rational function cross a horizontal asymptote infinitely many times? Either give an example or prove that this is not possible.
- (4) Write an equation for a rational function g , that is not a polynomial, that satisfies all of the following:
- $\lim_{x \rightarrow -\infty} g(x) = -\infty$.
 - $\lim_{x \rightarrow \infty} g(x) = \infty$.
 - The function g has no asymptotes.
- (5) For each item below, write an equation for a rational function that meets the given criteria. Sketch the graph of each function on labelled coordinate axes.
- (a) $\lim_{x \rightarrow \infty} f(x) = 0$ and the range of $f(x)$ includes only positive numbers.
- (b) $\lim_{x \rightarrow 5^+} h(x) = \infty$ and $\lim_{x \rightarrow 5^-} h(x) = \infty$.
- (c) The graph of the rational function has at least three disjoint components.