

**MAE 301/501 HOMEWORK-1 DUE AT THE BEGINNING OF CLASS ON
THURSDAY, SEPTEMBER 2**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

- (1) Give a clear proof of the Pythagorean theorem. For this problem, it is okay to consult outside sources. You should site any source you use. I am interested in how well you **explain** and understand the proof.
- (2) State and prove the law of cosines.
- (3) Given a right triangle, whose side lengths are relatively prime: $\gcd(a, b, c) = 1$. Prove that the hypotenuse is odd and that the two legs are of opposite parity: one odd and one even.
- (4) Take a to be the even leg of a right triangle, and b to be the odd leg. We can rewrite the Pythagorean identity as $(c - a)(c + a) = b^2$. Prove that the greatest common divisor of $(c - a)$ and $(c + a)$ is 1.
- (5) Using the fact that $(c - a)$ and $(c + a)$ are relatively prime, (have greatest common divisor of 1), prove that $(c - a)$ and $(c + a)$ are both perfect squares. That is, $c - a = r^2$, for some integer r , and $c + a = s^2$, for an integer s . Explain why it is necessary for the proof that $c - a$ and $c + a$ are relatively prime?
- (6) Now explain how you can choose relatively prime integers s and r to generate infinitely many primitive Pythagorean triples.