Practice Midterm 1

MAT 312

Feb 28, 2024

List of topics

1. GCD, LCM, Euclid algorithm. Writing $\gcd(a, b)$ as linear combination of a, b.
2. Mathematical induction
3. Primes, unique factorization. Finding all divisors of a number from its prime factorization.
4. Congruences and congruence classes. Solving linear congruences. Finding inverses in modular arithmetic.
5. Chinese remainder theorem. Euler's function.
6. Order of a number mod n. Fermat's little theorem and Euler's theorem.

Practice problems

Note that the actual exam will be much shorter than this collection of problems.

1. Find the greatest common divisor of 3 numbers: 28, 20, 70, and write it in the form $28s + 20t + 70r$.

2. A sequence x_n is defined by rules $x_1 = 5$, $x_{n+1} = 2x_n - 3$. Write down first seven terms; try to guess the formula for x_n and prove it using induction. [Hint: compare x_n with powers of 2.]

3. Show that if a is odd, then $\gcd(a, 2b) = \gcd(a, b)$.

4. Find the following inverses if they exist. If not, explain why.

(a) Inverse of 10 $\mod 33$

(b) Inverse of 29 $\mod 18$

(c) Inverse of 28 $\mod 18$

5. Find all solutions of the following congruences

(a) $8x \equiv 3 \mod 11$

(b) $18x \equiv 9 \mod 15$

(c) $18x \equiv 10 \mod 15$

6. The theory of biorhythms suggests that one's emotional and physical state is subject to periodic changes: 23-day physical cycle and a 28-day emotional cycle. (This is a highly dubious theory, but for this problem, let us accept it.) Janet found out that for her, January 1st, 2024 was the first day of both cycles. How many days will it take for her to achieve top condition on both cycles (which happens on 6th day of 23-day cycle and 7th day of 28-day cycle)? When will be the next time she achieves top condition in both cycles? (Note: first day is day 1, not day 0!)

7. Find the last two digits of 179^{2042}

8. (a) Show that for any n, $n^{121} - n$ is divisible by 7

(b) Show that for any n, $n^{121} - n$ is divisible by 1001. [Hint: you can use the fact that $1001 = 7 \cdot 11 \cdot 13$]