

MAT 312: HOMEWORK 9

1. Textbook, p. 252, problem 1
2. Textbook, p. 252, problem 2
3. A Universal Product Code (UPC) is a numeric code assigned to virtually all products sold in stores; you usually see it as a bar code, which you scan at the cash register. The UPC is a 12-digit numeric code $a_1 \dots a_{12}$; the 12 digits have to satisfy the condition below:

$$3a_1 + a_2 + 3a_3 + a_4 + \dots + a_{12} \equiv 0 \pmod{10}$$

The sum $3a_1 + a_2 + 3a_3 + a_4 + \dots + a_{12} \pmod{10}$ is called the *checksum*.

If the digits do not satisfy this condition, it can not be a valid UPC code — so probably there was an error when scanning the barcode and it needs to be re-scanned. For example, 380177-051136 is a valid UPC code (check!); however, if the first 3 was replaced by 8, this would change the checksum by $3 \times 5 = 15 \equiv 5 \pmod{10}$, so the new checksum wouldn't be zero.

Show that this scheme detects all single-digit errors. Does it detect all transpositions (i.e. interchange of two adjacent digits)?

4. (Optional problem) Construct a coding function $f: B^4 \rightarrow B^8$ that allows for detection of up to 3 bit errors and correction of a single bit error. [Hint: start with the Hamming (7,4) code discussed in class, and add another column to the generating matrix.]