MAT 312/AMS 351: HOMEWORK 7

ASSIGNED MAR 21, 2024

All textbook problems refer to *"Numbers, Groups, and Codes"*, by Humphreys and Prest, 2nd edition.

Unless stated otherwise, all numbers in this problem set are positive integers.

1. Textbook, p. 218, problem 1

2. Textbook, p. 219, problem 5

3. Let $G = \mathbb{Z}_n$ be the the group of congruence classes mod n with operation of addition. Show that if d is a divisor of n, i.e. $n = dk$, then G contains an element of order d.

4. Let $G = S_n$ be the group of all permutations of n elements, and let $H = \{s \in S_n \mid s(n) = n\}$.

(a) Show that H is a subgroup which is isomorphic to S_{n-1}.

(b) Let $s_1, s_2 \in S_n$ be in the same H-coset, i.e. $s_2 = s_1 h$ for some $h \in H$. Prove that then $s_1(n) = s_2(n)$.

(c) Show that conversely, if $s_1, s_2 \in S_n$ are such that $s_1(n) = s_2(n)$, then s_1, s_2 are in the same H-coset.

(d) Without using Lagrange's theorem, show that there are exactly n H-cosets in G.

5. Recall from last time the group R of all rotation symmetries of a regular tetrahedron, i.e. rotations of 3d space that preserve the regular tetrahedron.

Let v be one of the vertices of the tetrahedron. Denote $H_v = \{g \in R \mid g(v) = v\}$.

(a) Show that H_v is a subgroup.

(b) Show that $g_1, g_2 \in R$ are in the same H_v-coset if and only if $g_1(v) = g_2(v)$.

(c) Show that there are exactly 4 H_v-cosets, and they are naturally in correspondence with vertices of the tetrahedron.