MAT 312/AMS 351: HOMEWORK 6

ASSIGNED MAR 7, 2024

All textbook problems refer to *"Numbers, Groups, and Codes"*, by Humphreys and Prest, 2nd edition.

Unless stated otherwise, all numbers in this problem set are positive integers.

1. Textbook, p. 183, problem 1

2. Textbook, p. 183, problem 3

3. Show that the set A_n of all *even* permutations of n elements is a group. What about the odd permutations? do they form a group?

4. Given two elements x, y in a group G, define their *commutator* $[x, y]$ by $[x, y] = xyx^{-1}y^{-1}$.

(a) Show that x and y commute (i.e. $xy = yx$) if and only if $[x, y] = e$, where e is the group unit.

(b) Compute the commutator of permutations $x = (1\ 2\ 3\ 4\ 5)$, $y = (5\ 6\ 7\ 8\ 9)$, considered as elements of the group S_9.

5. Consider the set R of all rotation symmetries of a regular tetrahedron, i.e. rotations of 3d space that preserve the regular tetrahedron.

(a) How many elements are there in R?

(b) Prove that R is a group.

*(c) Every element of R permutes vertices of the tetrahedron and thus determines an element of S_4. Show that this allows one to identify R with the group A_4 of even permutations of 4 elements.