MAT 312/AMS 351: HOMEWORK 3

ASSIGNED FEB 7, 2024

All textbook problems refer to *"Numbers, Groups, and Codes"*, by Humphreys and Prest, 2nd edition.

Unless stated otherwise, all numbers in this problem set are positive integers.

1. Textbook, p. 48, problem 2

2. Textbook, p. 48, problem 3

3. Textbook, p. 48, problem 5 (hint: this is equivalent to $x^2 - 1 \equiv 0 \mod p$)

4. (a) Show that $10^n \equiv 1 \mod 9$ for any $n \geq 1$.
(b) Let x be a positive integer which is written in decimal form as $x = \overline{a_k \dots a_1 a_0}$ (thus, it has digit a_0 in ones place, digit a_1 in tens place, etc). Show that then $x \equiv a_0 + \cdots + a_k \mod 9$.
(c) Explain why a number is divisible by 9 if and only if sum of its digits is divisible by 9.

5. Can you repeat the same arguments as in the previous problem but for division by 11 rather than 9? (You will need to make some changes...)

6. Textbook, p. 58, problem 1

7. In a calendar of some ancient race, all months were exactly 30 days long; however, they used same weeks as we do. If in that calendar, first day of a certain month is Friday, how many weeks will pass before Friday will fall on the 13th day of a month? How frequently will such Friday the 13th repeat? [Hint: this can be rewritten as some congruence of the form $7x \equiv \dots \mod 30$ where x is the number of weeks.]