MAT 312/AMS 351: HOMEWORK 2

ASSIGNED JAN 31, 2024

All textbook problems refer to '*Numbers, Groups, and Codes*", by Humphreys and Prest, 2nd edition.

Unless stated otherwise, all numbers in this problem set are positive integers.

1. Textbook, p. 23, problem 1

2. Textbook, p. 23, problem 3

3. Textbook, p. 23, problem 8

4. Explain what is wrong with the following "proof".

"Theorem". All pens have the same color ink.

"Proof." We will argue by induction on n that given any set of n pens, they all have the same color ink.

If $n = 1$, this is clear since we're talking about a single pen. So assume that any set of k pens all have the same color ink, and consider a set S of $k+1$ pens. If we remove the first pen from S we are left with a set of k pens, which by induction all have the same color ink. On the other hand, if we remove the last pen from S we get another set of k which also have the same color ink. Therefore all the pens in S have the same color ink, which completes the induction step.

5. Let $a = p_1^{n_1} \dots p_r^{n_r}$, where p_i are distinct primes. Show that then a has $(n_1+1)(n_2+1)\dots(n_r+1)$ different divisors.

6. Show the following:

(a) $2^{3m}+1$ is divisible by 2^m+1 and thus can not be prime. [Hint: $x^3+1 = (x+1)(x^2-x+1)$.]

(b) For any odd l, $2^{lm}+1$ is divisible by 2^m+1.

(c) Deduce from this that if 2^n+1 is prime, then n must itself be a power of 2. [Note that it is necessary, but not sufficient condition.]

7. Textbook, p. 36, problem 9