MAT 303: MIDTERM 2 REVIEW AND PRACTICE PROBLEMS

SPRING 2024

Midterm rules

Midterm will be on Fri, April 5, 2024 in class. It will be a closed book exam: no books, notes, or calculators. There will be no more than 4 problems (but some of them might be multi-part).

Please write full solutions (not just answers!). Solutions should be written so that they are easy to read, understand and follow. Anything that is not part of your final solution (e.g. preliminary computations you later abandoned) should be crossed out.

Sections covered

Exam covers the following textbook sections:
3.1 - 3.6, 4.1 - 4.2, 5.1, 5.2, 5.5
with the following exceptions:

(1) In 4.2, we haven't discussed linear differential operators - only basic eleimination
(2) In 5.1, we skipped discussion of Wronskian of a set of solutions of a system of differential equations

Earlier material might be necessary for solving problems on the topics above, but there will be no problems specifically about earleir topics.

Practice problems

Warning: this is just one possible selection of problems — there is no promise that the actual exam will contain the exact same types of problems!

This set of practice problems is significantly longer than the actual exam, to give you more opportunities to practice.

(1) Find the general solution of the following differnetial equation

$$y^{(3)} - 2y'' + 2y' = 6\sin(3x).$$

(2) Solve the following initial value problem:

$$y'' - 3y' + 2y = x + e^x, \quad y(0) = 0, y'(0) = 1$$

(3) Use the method of variation of parameters to find a particular solution of the equation

$$y'' + y = \frac{1}{\cos(x)}$$

(it is ok to write an answer in the form that includes indefinite integrals).

(4) Find the general solution of the equation

$$y^{(4)} - 2y'' + y = 0$$

(5) Use the method of elimination to find the general solution of the following system of differential equations:

$$x'' = 6x + 2y$$
$$y'' = 3x + 7y$$

where x, y are functions of variable t and prime stands for derivative with respect to t.

(6) Consider the following system of differential equations (note that the first equation is a second order equation!)

$$x'' = -x + y$$
$$y' = -3x - x' + 3y$$

where x, y are functions of variable t and prime stands for derivative with respect to t.

(a) Convert it to a system of linear **first order** differential equations and write it in the form $\mathbf{x}' = A\mathbf{x}$ for some vector $\mathbf{x}$ (depending on t) and matrix A.
(b) Find the eigenvalues and eigenvectors of A
(c) Write the general solution of the system from part (a).

(7) Find a solution of the following initial value problem

$$\mathbf{x}' = \begin{pmatrix} 1 & -5 \\ 1 & 5 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$