MAT 303: FINAL REVIEW AND PRACTICE PROBLEMS

SPRING 2024

Final rules

Final will be on Wed, May 15, in class. You will be allowed to bring one "cheat sheet": one sheet of notes (ltter size, two-sided). Other than that, no notes book, or calculators may be used.

Please write full solutions (not just answers!). Solutions should be written so that they are easy to read, understand and follow. Anything that is not part of your final solution (e.g. preliminary computations you later abandoned) should be crossed out.

Sections covered

Exam covers the following textbook sections:
1.1 –1.6, 2.1 – 2.4, 3.1 – 3.6, 4.1 - 4.2, 5.1, 5.2, 5.5, 5.6, 7.1–7.3, 8.1
with the following exceptions:

(1) In 2.2, we haven't discussed section about bifurcation
(2) In 2.3, we haven't discussed case when resistance is proportional to square of air velocity.
(3) In 2.4, we haven't talked about estimating the error of Euler method
(4) In 4.2, we haven't discussed linear differential operators - only basic elimination
(5) In 5.1, we skipped discussion of Wronskian of a set of solutions of a system of differential equations

Practice problems

This practice problems cover the material after Midterm 2. For practice problems on earlier material, please check practice midterms 1 and 2.

(1) Consider the matrix

$$A = \begin{pmatrix} -3 & 3 & 0 \\ 2 & -4 & 2 \\ 0 & 3 & -3 \end{pmatrix}$$

It is known that the eigenvalues of this matrix are 0, -3, -7.

(a) Find the eigenvectors of this matrix

(b) Find the general solution of the differential equation $X' = AX$

(2) Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$$

(a) Find the eigenvalues and eigenvectors of this matrix

(b) Find the general solution of the differential equation $X' = AX$

(3) (a) Compute the matrix exponent e^{tA}, where A is the 2×2 matrix below

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

(b) Solve the initial vlaue problem

$$X' = AX, \qquad X(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(4) Use Laplace transform to solve:

$$x'' + 3x' + 2x = t, \quad x(0) = 1, x'(0) = 0$$

(5) (a) Compute Laplace transform of the function

$$f(t) = \sqrt{t}e^{-2t}$$

(b) Compute inverse Laplace transform of the following function

$$F(s) = \frac{1}{s^2(s^2+1)}$$

(6) Use the power series method to find a solution of the initial value problem

$$y' + (2x+1)y = 0, \qquad y(0) = 2.$$

No credit for doing this problem using other methods