

Problem Set 4

Disclaimer For open-ended problems, part of the problem is to give a precise formulation. Especially for the problems in Part II, you should do as much of the problems as is useful to you. For each problem, it is important you understand how to verify all details. However, if you are pressed for time, you may write-up only the most important steps, instead of every detail.

Late homework policy. Late work will be accepted only with a medical note or for another approved reason.

Cooperation policy. You are strongly encouraged to work with others, but the final write-up must be entirely your own and based on your own understanding.

Part I. These problems are from the textbook. You are expected to read *all* the problems from the sections of the textbook covered that week. You are asked to write-up and turn-in only the problems assigned below.

Part II. These problems are not necessarily from the textbook. Often they will be exercises in commutative algebra, category theory, homological algebra or sheaf theory.

Part I(25 points)

- (a) (5 points) p. 91, Section II.3, Problem 3.4
- (b) (10 points) p. 91, Section II.3, Problem 3.5 (a) and (b) (think about (c))
- (c) (10 points) p. 91, Section II.3, Problem 3.7

Part II(25 points)

Problem 1(5 points) Read through Exercise II.3.13 on p. 93 of the textbook. Write out 1 or 2 parts of your choice. (Part (f) is fun.)

Problem 2(10 points) Work through Exercise II.3.19 on p. 94 of the textbook. You need not write up every step, but write up the main steps. You will need to read through and understand Exercises II.3.16–II.3.18. You may also want to look up “generic flatness” or “generic freeness” in a commutative algebra textbook.

Problem 3(10 points) Work through Exercise II.3.22 on p. 95 of the textbook. You need not write up every step, but write up the main steps. You will need to read through and understand Exercise II.3.20. You may also want to look up Krull’s Hauptidealsatz, systems of parameters, the going-up theorem and Noether’s normalization theorem.

Extra credit(5 points) Give an example of a Noetherian scheme with infinite dimension.