Guidorizzi II §11.3 p.204

10. β é um plano tangente aos gráficos de $f(x,y) = 2 + x^2 + y^2$ e $g(x,y) = -x^2 - y^2$. Mostre que $a^2 + b^2 = 1$, sendo $(a, b, f(a,b))$ o ponto em que β tangencia o gráfico de f.

Guidorizzi §11.3 Exc. 10 p. 205:

$$P = (a, b, \overbrace{f(a,b)}^{2+a^2+b^2}) \qquad S = gr(f)$$

$$\tilde P = (\tilde a, \tilde b, \overbrace{g(\tilde a,\tilde b)}^{-\tilde a^2-\tilde b^2}) \qquad \tilde S = gr(g)$$

$$T_P S = \{ (x,y,z)\in\mathbb{R}^3 \mid z - \underbrace{z_0}_{f(a,b)} = \underbrace{f_x(a,b)}_{2a}(x-a) + \underbrace{f_y(a,b)}_{2b}(y-b)\}$$

β

mesmo plano $\Rightarrow$ mesmas inclinações

$$T_{\tilde P}\tilde S = \{ z - \underbrace{z_0}_{g(\tilde a,\tilde b)} = 2a(x-\tilde a) + 2b(y-\tilde b)\}$$

Condição 1:

$(0,0,z)\in T_P S$: $$z = 2 + \overbrace{a^2+b^2 + \underbrace{2a(0-a)}_{-2a^2} + \underbrace{2b(0-b)}_{-2b^2}}^{-a^2-b^2}$$

$$\|$$

$(0,0,z)\in T_{\tilde P}\tilde S$: $$z = -\tilde a^2 - \tilde b^2 + \underbrace{2a(0-\tilde a)}_{-2a\tilde a} + \underbrace{2b(0-\tilde b)}_{-2b\tilde b}$$

$$\Leftrightarrow \boxed{2 = a^2+b^2-\tilde a^2-\tilde b^2 - 2a\tilde a - 2b\tilde b} \Rightarrow 2 = 2a^2+2b^2$$

$$\Downarrow$$

$$\boxed{1 = a^2+b^2}$$ q.e.d.

Condição 2: vetores normais iguais

$$N_P^f = N_{\tilde P}^g$$

$$\begin{pmatrix}2a\\2b\\-1\end{pmatrix} = \begin{pmatrix}f_x(a,b)\\f_y(a,b)\\-1\end{pmatrix} = \begin{pmatrix}g_x(\tilde a,\tilde b)\\g_y(\tilde a,\tilde b)\\-1\end{pmatrix} = \begin{pmatrix}-2\tilde a\\-2\tilde b\\-1\end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\begin{aligned}a &= -\tilde a\\ b &= -\tilde b\end{aligned}}$$