MAT 203 MIDTERM I

MONDAY FEBRUARY 23, 2026
5:00–6:20PM

Name: ______________________________ ID: ______________

Instructions.

(1) Fill in your name and Stony Brook ID number.

(2) This exam is closed-book and closed-notes; no electronic devices.

(3) You have 80 minutes to complete this exam.

(4) You must justify all your answers and show all your work. Even a correct answer without any justification will result in no credit.

1. (a) (2 pts) Let $\vec{u} = \langle 4, 5\rangle$ and $\vec{w} = \langle -2, 6\rangle$. Find the vector $\vec{v}$ such that $\vec{u} + \vec{v} = \vec{w}$.

Solution. We solve the equation to get $\vec{v} = \vec{w} - \vec{u} = \langle -2, 6\rangle - \langle 4, 5\rangle = \langle -6, 1\rangle$. □

(b) (8 pts) Let $\vec{u} = \langle 1, 1, 1\rangle$. Find a unit vector $\vec{v}$ such that $\vec{u} \cdot \vec{v} = 0$.

Solution. If $\vec{v} = \langle v_1, v_2, v_3\rangle$, then $\vec{u} \cdot \vec{v} = 0$ is equivalent to the equation

$$v_1 + v_2 + v_3 = 0.$$

We see that the vector $\langle 0, 1, -1\rangle$ satisfies this equation. Now, to find a unit vector we normalize the vector to get

$$\vec{v} = \frac{\langle 0, 1, -1\rangle}{||\langle 0, 1, -1\rangle||} = \frac{\langle 0, 1, -1\rangle}{\sqrt{2}} = \left\langle 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\rangle.$$

□

2. (a) (5 pts) Find a parametrization of the line passing through the points $P = (-1, 2, 4)$ and $Q = (5, -3, -5)$.

Solution. A direction vector is given by $\vec{PQ} = \langle 5 - (-1), -3 - 2, -5 - 4 \rangle = \langle 6, -5, -9 \rangle$, and therefore a parametrization is given by

$$\vec{\gamma}(t) = \langle -1, 2, 4 \rangle + t \langle 6, -5, -9 \rangle .$$

□

(b) (5 pts) Find a parametrization of the plane described by the equation $2x - y + 4z + 1 = 0$.

Solution. To find a parametrization we will find a point and two non-parallel vectors inside the plane. We begin by finding 3 points. We see that for example $P = (0, 1, 0)$, $Q = (1, 3, 0)$ and $R = (0, 5, 1)$ are all points that belong to the plane. We then find two non-parallel vectors in the plane by taking differences:

$$\vec{PQ} = \langle 1, 2, 0 \rangle$$
$$\vec{PR} = \langle 0, 4, 1 \rangle$$

A parametrization is then given by

$$\vec{r}(s, t) = \langle 0, 1, 0 \rangle + s \langle 1, 2, 0 \rangle + t \langle 0, 4, 1 \rangle .$$

□

3. (10 pts) Find the center and the three radii of the hyperboloid described by the equation

$$x^2 - 2x - 4y^2 - 8y - z^2 + 2z = 8.$$

Solution. We complete the squares:

$$\begin{aligned} x^2 - 2x &= (x-1)^2 - 1 \\ -4y^2 - 8y &= -4(y^2 + 2y) = -4(y+1)^2 + 4 \\ -z^2 + 2z &= -(z^2 - 2z) = -(z-1)^2 + 1 \end{aligned}$$

Therefore the equation $x^2 - 2x - 4y^2 - 8y - z^2 + 2z = 8$ is equivalent to

$$\begin{aligned} &((x-1)^2 - 1) + (-4(y+1)^2 + 4) + (-(z-1)^2 + 1) = 8 \\ \iff &(x-1)^2 - 4(y+1)^2 - (z-1)^2 = 4 \\ \iff &\frac{(x-1)^2}{2^2} - (y+1)^2 - \frac{(z-1)^2}{2^2} = 1. \end{aligned}$$

We now see that the center is $(1, -1, 1)$, and the three radii are $a = 2$, $b = 1$, and $c = 2$. □

4. Consider the curve S that is parametrized by $\vec{r}(t) = \langle t, t, t^2 \rangle$.

(a) (5 pts) Compute $\frac{d}{dt}(\vec{r}(t) \times \vec{r}\,'(t))$.

Solution. First we have $\vec{r}\,'(t) = \langle 1, 1, 2t \rangle$. The cross product is now computed by

$$\vec{r}(t) \times \vec{r}\,'(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & t & t^2 \\ 1 & 1 & 2t \end{vmatrix} = \langle 2t^2 - t^2, -(2t^2 - t^2), t - t \rangle = \langle t^2, -t^2, 0 \rangle .$$

Finally

$$\frac{d}{dt}(\vec{r}(t) \times \vec{r}\,'(t)) = \frac{d}{dt} \langle t^2, -t^2, 0 \rangle = \langle 2t, -2t, 0 \rangle .$$

□

(b) (5 pts) Compute $\int_0^1 ||\vec{r}(t) \times \vec{r}\,'(t)||\, dt$.

Solution. From part (a) we have $\vec{r}(t) \times \vec{r}\,'(t) = \langle t^2, -t^2, 0 \rangle$. The magnitude is given by

$$||\vec{r}(t) \times \vec{r}\,'(t)|| = ||\langle t^2, -t^2, 0 \rangle|| = \sqrt{2t^4} = \sqrt{2}t^2,$$

and so

$$\int_0^1 ||\vec{r}(t) \times \vec{r}\,'(t)||\, dt = \int_0^1 \sqrt{2}t^2\, dt = \sqrt{2} \left[\frac{t^3}{3} \right]_0^1 = \frac{\sqrt{2}}{3}.$$

□