Print your name: ______________________________

Answer each question completely. You must justify your answers to get credit. Even a correct answer with no justification will get no credits. Each problem is worth 5 points.

1. Find a formula for the general term a_n of the following sequence, assuming that the pattern of the first few terms continues. (Assume that n begins at 1.)

$$\left\{\frac{1}{4}, -\frac{4}{5}, \frac{9}{6}, -\frac{16}{7}, \frac{25}{8}, \dots\right\}.$$

Solution. The first few terms are $a_1 = \frac{1^2}{1+3}$, $a_2 = -\frac{2^2}{2+3}$, $a_3 = \frac{3^2}{3+3}$, and $a_4 = -\frac{4^2}{4+3}$. The general term is given by

$$a_n = (-1)^{n+1}\frac{n^2}{n+3}.$$

□

2. Determine if the following sequence converges or diverges

$$a_n = \arctan\left(1 + \frac{n}{n^2+3}\right) + e^{\frac{1}{n}}$$

Solution. Since both $\arctan x$ and e^x are continuous functions (everywhere), we have

$$\lim_{n\to\infty} \arctan\left(1 + \frac{n}{n^2+3}\right) + e^{\frac{1}{n}} = \arctan\left(\lim_{n\to\infty}\left(1 + \frac{n}{n^2+3}\right)\right) + e^{\lim_{n\to\infty}\frac{1}{n}}.$$

The limit inside of arctan is

$$\lim_{n\to\infty}\left(1 + \frac{n}{n^2+3}\right) = 1 + \lim_{n\to\infty}\frac{n}{n^2+3} = 1 + \lim_{n\to\infty}\frac{\frac{1}{n}}{1+\frac{3}{n}} = 1 + 0,$$

therefore the above limit is equal to $\arctan(1) + e^0 = \frac{\pi}{4} + 1$. The sequence is therefore convergent, because the limit exists. □