MAT 127 Lec 16 Tue 22 Oct

Recall:

General ODE:

Equation involving a function $y=y(x)$, one or more of its derivatives, and x.

The order of an ODE is the highest order of a derivative that occurs.

Ex: Verify that $y=2Ce^{-2t}+e^t$ is a solution to $y'+2y=3e^t$ & solve the IVP

$$\boxed{y'+2y=3e^t, \quad y(0)=3}$$

We first verify: $y'=-4Ce^{-2t}+e^t$

$$LHS=(\cancel{-4Ce^{-2t}}+e^t)+2(\cancel{2Ce^{-2t}}+e^t)$$

$= 3e^t = \text{RHS}$ ✓

Slope fields & Euler's method

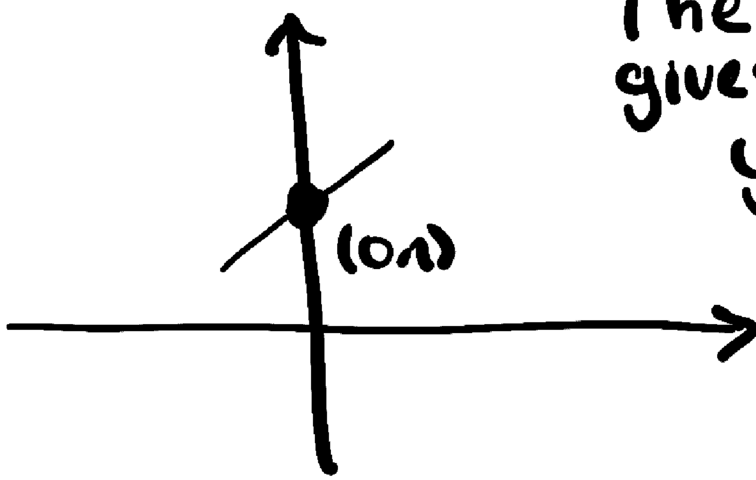
(§7.2 Stewart)

In general, can't solve ODE's unless they are of specific forms. Can still learn a lot about them using a graphical approach (slope field) or numerical approach (Euler's method).

Ex: Suppose we need to sketch the graph of the solution to the IVP $y' = x + y$, $y(0) = 1$.

The equation tells us what the <u>slope</u> is of y at the point $(x, y(x))$.

The initial cond. tells us that the solution passes through $(0,1)$ $(y(0)=1)$.



The ODE then gives

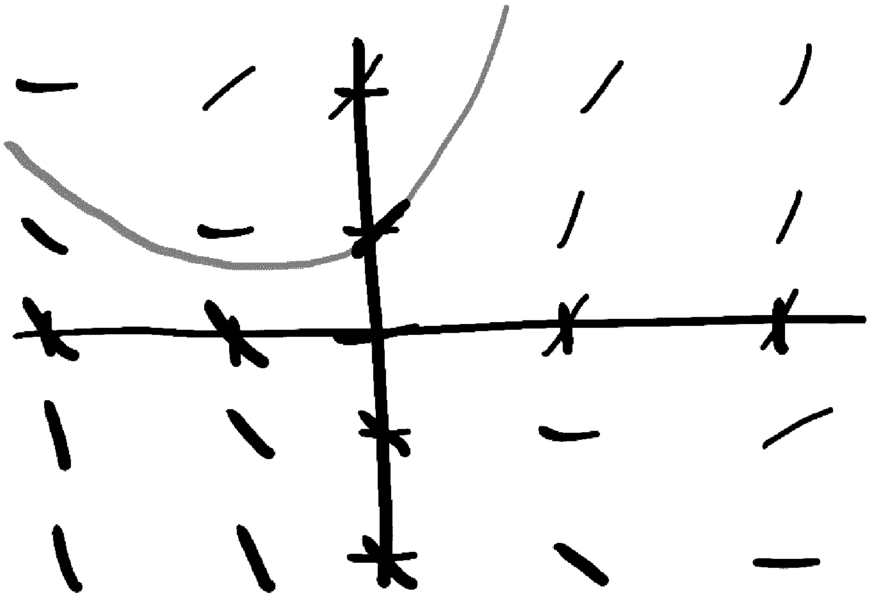
$$y' = x + y = 0 + 1 = 1$$

at this pt.

So the solution, whatever it is, will have slope 1 at $(0,1)$.

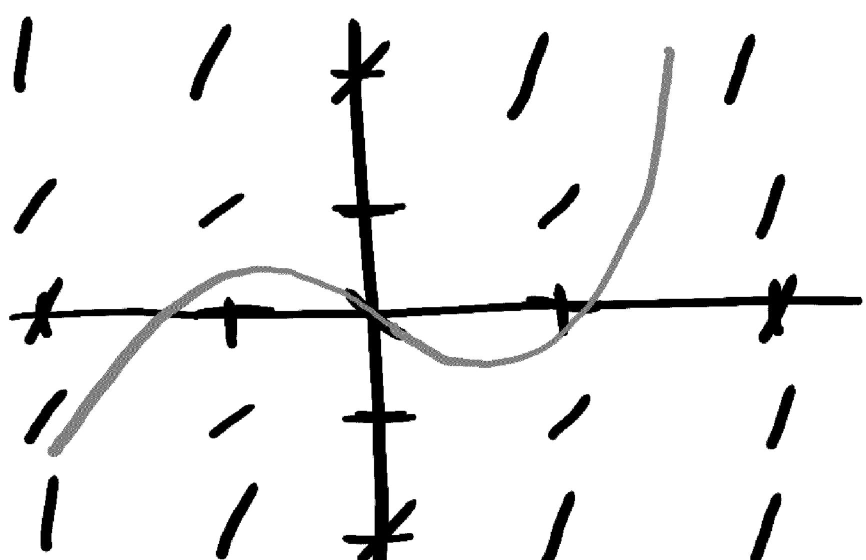
Sketch of more slopes:

$y \backslash x$	-2	-1	0	1	2
-2	-4	-3	-2	-1	0
-1	-3	-2	-1	0	1
0	-2	-1	0	1	2
1	-1	0	1	2	3
2	0	1	2	3	4



Ex: Sketch slope field of
$y' = x^2 + y^2 - 1$ & sketch the
graph of the solution with $y(0) = 0$

y \ x	-2	-1	0	1	2
-2	7	4	3	4	7
-1	4	1	0	1	4
0	3	0	-1	0	3
1	4	1	0	1	4
2	7	4	3	4	7

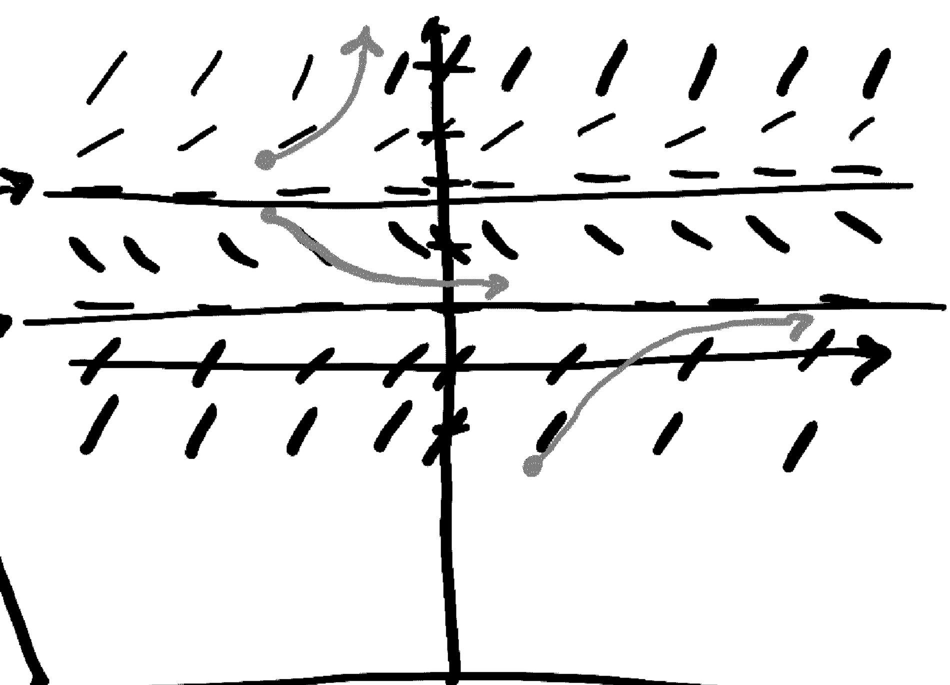


<u>Ex</u>: $y' = (y-1)(y-3)$.

Let's sketch slope field.

Note that the RHS does <u>not</u> depend on x, so slope only dep on y.

y	y'
-1	8
0	3
1	0
2	-1
3	0
4	3
5	8

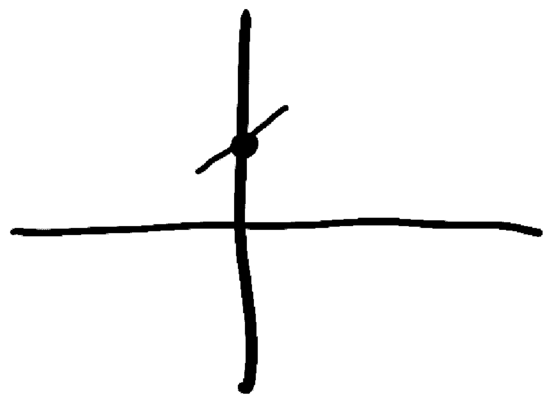


Equilibrium solutions $y=1$ and $y=3$

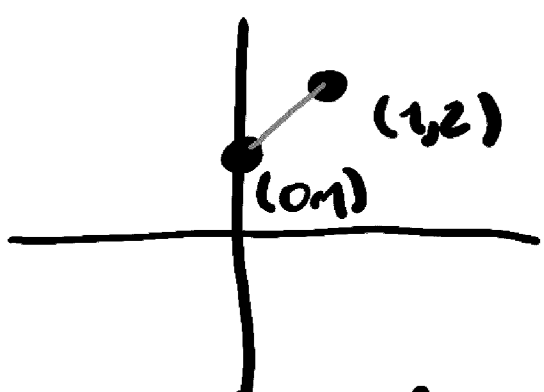
<u>Euler's method</u>

Consider $y' = x+y$, $y(0)=1$ again.

We already know that $y'(0) = 1$ by plugging in $(x,y) = (0,1)$ into the ODE.



Now let's approximate the solution by taking a small step in the direction of this slope.

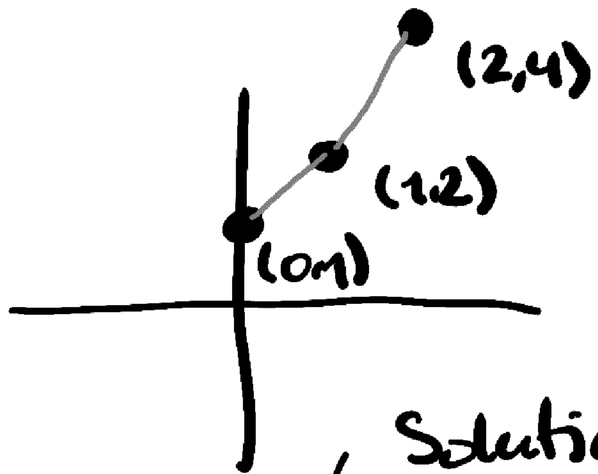


Took a step of size 1 (from $x=0$ to $x=1$)

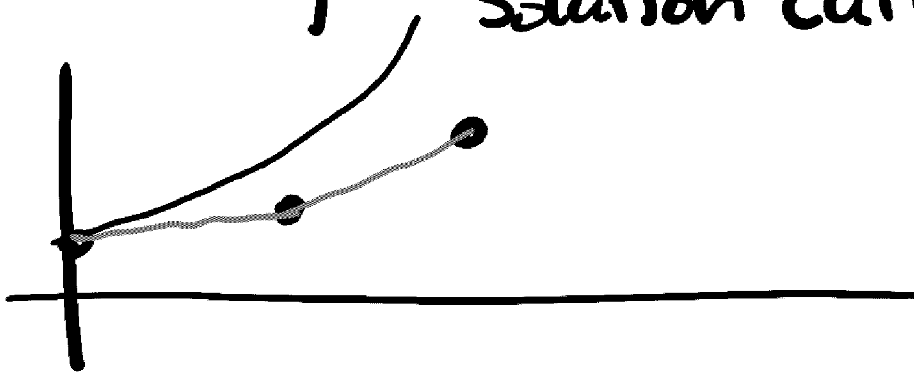
Now adjust slope according to

the ODE $y' = x + y$.

New slope $= y'(1) = 1 + 2 = 3$



Solution curve



Can use smaller step sizes to get better approximation