

Admin stuff: (See syllabus at Brightspace)

Class time: TR 5:00-6:20pm

Me: Johan Asplund (Math 3-116)
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office hours: TR 4-5 pm

Textbook: Stewart 5th ed.

Webpage (notes):

math.stonybrook.edu/~jasplund/mat127_fall24

Homework:

- Weekly WebAssign assignments
- 1-2 problem in-class quiz
on Lec 3 and (almost) every other
lecture.

↳ Gradescope.

MAKE SURE YOU HAVE ACCESS TO BOTH
WEBASSIGN & GRADESCOPE!
(else contact me)

Grading: Homework: 20%

Midterms (Oct 1, Nov 6): 20% each

Final (Dec 11): 40%

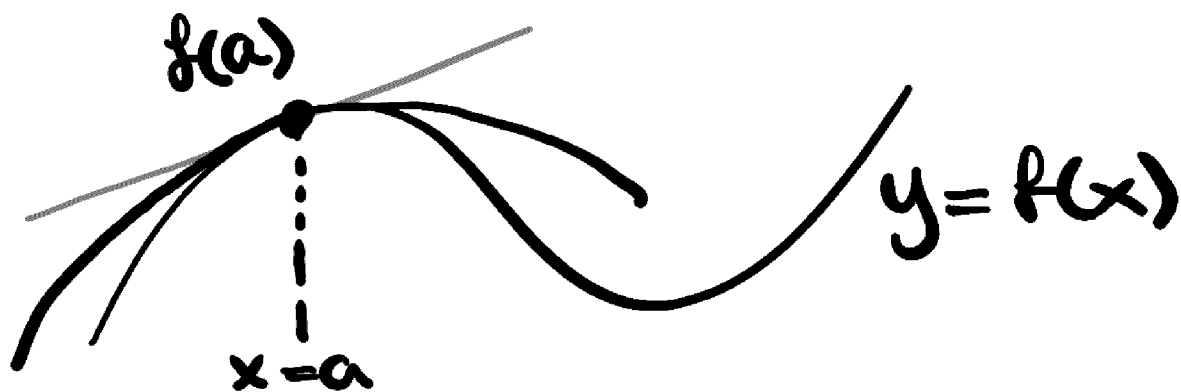
§ Overview of course

Main topics:

① Infinite Series (Taylor series)

② Differential equations.

① Calc A: Derivatives



Linear approx at $x = a$.

$$\boxed{f(x) \approx f(a) + f'(a)(x-a)}$$

Taylor series: Approximate $f(x)$

better by involving more derivatives!

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

"Parabolic approx at $x=a$ "

Use infinite number of terms to not only get an approx, but equality!

→ need to develop tools to deal w/ infinite sums.

② Ordinary differential equations (ODEs)

Ex Find function $f(x)$

Such that $f'(x) = f(x)$.

We see that $f(x) = e^x$ works.

What about $f'(x) \cdot f(x) = 2$?

- Develop tools to solve these.
- Applications:
e.g. modeling populations

§ Sequences (Stewart 8.1)

An ordered list of "things".

usually numbers

Ex:

① 1, 2, 3, 4, ...

② $-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \dots$

③

14, 18, 23, 28, 34, 42, 50, 59, ...

Def:

- An (infinite) sequence is an ordered list of numbers.

$$a_1, a_2, a_3, \dots = \{a_n\}_{n=1}^{\infty}$$

1st term 2nd term 3rd term

- n is called the index.
 - a_n is the n^{th} term
-

EX ① $a_n = n$ for all $n \geq 1$

② $a_n = (-1)^n \cdot \frac{1}{n}$ for all $n \geq 1$

③ $a_n = n^{\text{th}}$ numbered stop
on NYC uptown 1 train.

④ $\frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}, \frac{7}{36}, \dots$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

Numerators increase by 1.

So n^{th} numerator is $n+2$.

Denominators are squares.

$2^2 = 4, 3^2 = 9$ etc.

so $a_n = \frac{n+2}{(n+1)^2}$.

Sometimes it's hard or difficult
to find a defining equation
for a_n .

Ex: Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, ...



$$2 = 1 + 1 \quad 3 = 2 + 1 \quad 5 = 3 + 2$$

In general:

$$\boxed{a_n = a_{n-1} + a_{n-2}} \text{ for all } n \geq 3$$

Recurrence relation

Ex: Consider the sequence

given by $a_n = \frac{n}{n+1}$.

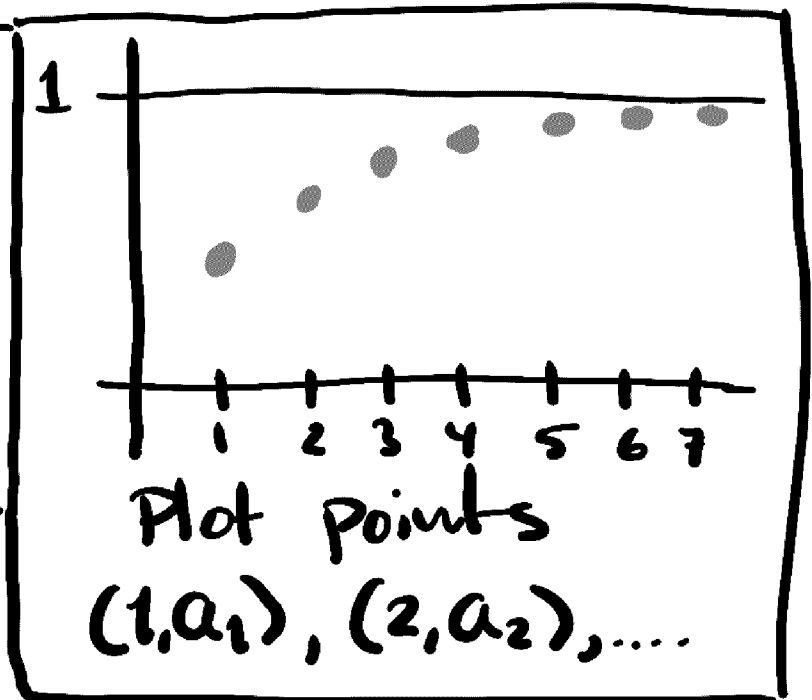
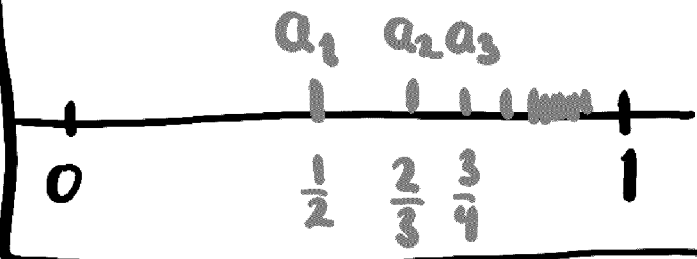
First few terms:

$$a_1 = \frac{1}{1+1} = \frac{1}{2} \quad \left| \quad a_4 = \frac{4}{5}$$

$$a_2 = \frac{2}{2+1} = \frac{2}{3} \quad \left| \quad a_5 = \frac{5}{6}$$

$$a_3 = \frac{3}{4}$$

on real line:



Points seem to go closer and closer to 1.

Note: n^{th} term is $\frac{n}{n+1}$ &

it can never be ≥ 1
because

denominator $>$ numerator.

How close is a_n to 1?

$$1 - a_n = 1 - \frac{n}{n+1} = \frac{n+1}{n+1} - \frac{n}{n+1} = \frac{1}{n+1}.$$

When n is big, this difference is very small: $\frac{1}{10,001} = 1 - a_{10,000}$

We say that $\frac{n}{n+1}$ approaches 1 as n becomes large.

We write: $\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1$

Def A sequence $\{a_n\}_{n=1}^{\infty}$ has the limit L , written as

$\lim_{n \rightarrow \infty} a_n = L$ or $a_n \rightarrow L$ as $n \rightarrow \infty$

if we can make the terms a_n as close to L as we like by taking n sufficiently large.

If $\lim_{n \rightarrow \infty} a_n$ exists we say that $\{a_n\}$ converges, or that it is convergent. Otherwise $\{a_n\}$ diverges, or is divergent.

Note that this is similar to limits of functions $\lim_{x \rightarrow \infty} f(x)$.

Thm: If $\lim_{x \rightarrow \infty} f(x) = L$ and $f(n) = a_n$ where n is an integer, then $\lim_{n \rightarrow \infty} a_n = L$.

Ex: We know $\lim_{x \rightarrow \infty} \frac{1}{x^r} = 0$ for $r > 0$. Therefore, with $f(x) = \frac{1}{x^r}$ the sequence $a_n = f(n) = \frac{1}{n^r}$ converges to 0 as $n \rightarrow \infty$.

$$\lim_{n \rightarrow \infty} \frac{1}{n^r} = 0 \text{ for } r > 0$$

Limit laws

$\{a_n\}$, $\{b_n\}$ two sequences and
 $c = \text{constant}$.

$$1. \lim_{n \rightarrow \infty} (a_n \pm b_n) = \lim_{n \rightarrow \infty} a_n \pm \lim_{n \rightarrow \infty} b_n$$

$$2. \lim_{n \rightarrow \infty} c \cdot a_n = c \cdot \lim_{n \rightarrow \infty} a_n$$

$$3. \lim_{n \rightarrow \infty} c = c$$

$$4. \lim_{n \rightarrow \infty} (a_n b_n) = \left(\lim_{n \rightarrow \infty} a_n \right) \left(\lim_{n \rightarrow \infty} b_n \right)$$

$$5. \lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\lim_{n \rightarrow \infty} a_n}{\lim_{n \rightarrow \infty} b_n} \text{ if } \lim_{n \rightarrow \infty} b_n \neq 0.$$

$$6. \lim_{n \rightarrow \infty} a_n^p = \left(\lim_{n \rightarrow \infty} a_n \right)^p \quad p > 0. \\ \text{and } a_n > 0.$$

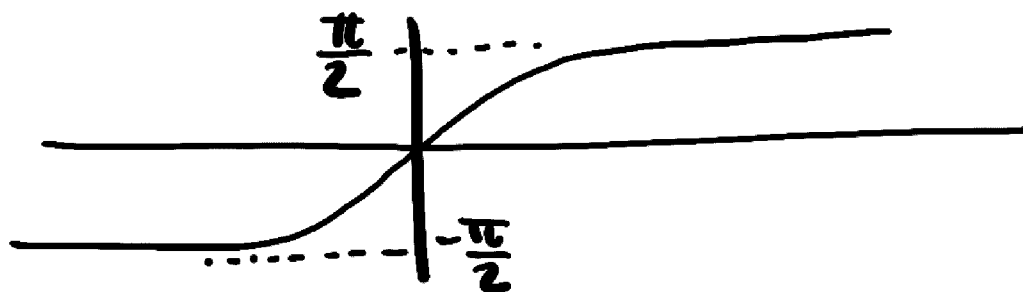
Ex: $a_n = \frac{n^2 + 3}{27n^2 - \pi + n}$. Use same technique as for functions:

Since it's the quotient of two polynomials we divide both the numerator and denominator by the highest power of n .

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{n^2 + 3}{27n^2 - \pi + n} &= \lim_{n \rightarrow \infty} \frac{\frac{n^2}{n^2} + \frac{3}{n^2}}{\frac{27n^2}{n^2} - \frac{\pi}{n^2} + \frac{n}{n^2}} \\ &= \lim_{n \rightarrow \infty} \frac{1 + \frac{3}{n^2}}{27 - \frac{\pi}{n^2} + \frac{1}{n}} = \frac{\lim_{n \rightarrow \infty} 1 + \lim_{n \rightarrow \infty} \frac{3}{n^2}}{\lim_{n \rightarrow \infty} 27 - \lim_{n \rightarrow \infty} \frac{\pi}{n^2} + \lim_{n \rightarrow \infty} \frac{1}{n}} \\ &= \frac{1 + 0}{27 - 0 + 0} = \frac{1}{27}.\end{aligned}$$

Ex: $a_n = \sqrt{\arctan(n)}$.

$a_n = f(n)$, $f(x) = \arctan(x)$.



Remember $\lim_{x \rightarrow \infty} \arctan(x) = \frac{\pi}{2}$

So

$$\lim_{n \rightarrow \infty} \sqrt{\arctan(n)} = \lim_{n \rightarrow \infty} (\arctan(n))^{1/2}$$
$$= \left(\lim_{n \rightarrow \infty} \arctan(n) \right)^{1/2} = \left(\frac{\pi}{2} \right)^{1/2} = \sqrt{\frac{\pi}{2}}.$$

Ex: Find $\lim_{n \rightarrow \infty} \frac{\ln n}{n}$.

We have $a_n = \frac{\ln n}{n} = f(n)$

for the real-valued function

$$f(x) = \frac{\ln x}{x}.$$

Consider $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$ and notice

$$\underbrace{\lim_{x \rightarrow \infty} \ln x = \infty}_{\text{numerator}}, \quad \underbrace{\lim_{x \rightarrow \infty} x = \infty}_{\text{denominator}}$$

Remember l'Hôpital's rule
from Calculus A:

$\lim_{x \rightarrow \infty} f(x)$ is of the indeterminate
form $\frac{\infty}{\infty}$ so

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} \ln x}{\frac{d}{dx} x}$$

$$= \lim_{x \rightarrow \infty} \frac{1/x}{1} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

Hence $\boxed{\lim_{u \rightarrow \infty} \frac{\ln u}{u} = 0}$.
