WEEK 4 HOMEWORK

From Gathmann's text, do 3.12, 3.20, 3.22, and 3.23.

Extra problem. Let $\mathscr{F}$ be a sheaf of abelian groups on a topological space X. Let $s \in \mathscr{F}(U)$ be a section of $\mathscr{F}$ over an open subset $U \subseteq X$.

(a) Define the *support* of s,

$$\operatorname{Supp} s = \big\{\, x \in U \;\big|\; s_x \neq 0 \text{ in the stalk } \mathscr{F}_x \,\big\},$$

to be the set of points $x \in X$ where the image s_x in the stalk $\mathscr{F}_x$ is nonzero. Show that $\operatorname{Supp} s$ is a closed subset of U.

(b) Define the *support* of $\mathscr{F}$,

$$\operatorname{Supp} \mathscr{F} = \big\{\, x \in X \;\big|\; \mathscr{F}_x \neq 0 \,\big\},$$

to be the set of points in X where the stalk of $\mathscr{F}$ is nonzero. Show by example that $\operatorname{Supp} \mathscr{F}$ need *not* be closed in X.