MATH 546, HOMEWORK 2

These problems are from Krantz, *Function Theory of Several Complex Variables.*

1. Let D be a connected Reinhardt domain. Prove that if f is holomorphic on D, then f has a Laurent expansion that converges to f uniformly on compact subsets of D.

2. Examine the domains of the convergence (in $\mathbb{C}^2$) for the following two series:

$$\sum_{j=0}^{\infty}(z_1+z_2)^j, \quad \sum_{j=0}^{\infty}(z_1z_2)^j.$$

Verify that they are logarithmically convex and complete Reinhardt.

3. Consider the circular domain

$$\begin{aligned} D = &\left\{ (z_1,z_2)\in\mathbb{C}^2 \;\middle|\; 1/4<|z_2|<1, |z_1|<1 \right\} \\ &\cup\left\{ (z_1,z_2)\in\mathbb{C}^2 \;\middle|\; |z_1|<1/2, |z_2|<1/2 \right\}. \end{aligned}$$

Let $\tilde{D}$ be the smallest complete and logarithmically convex Reinhardt domain containing D. Compute $\log\|D\|$ and $\log\|\tilde{D}\|$.

4. Let $\Delta^n \subseteq \mathbb{C}^n$ be the standard polydisk, and let G be the group of biholomorphic mappings from Δ^n to itself that fix the origin.

(a) Show that G has $n!$ many connected components.

(b) Show that the connected component of the identity consists of all maps

$$(z_1,\ldots,z_n)\mapsto(e^{i\theta_1}z_1,\ldots,e^{i\theta_n}z_n),$$

with $0\le\theta_1,\ldots,\theta_n<2\pi$.

Due on Tuesday, September 15.