MAT 562: Symplectic Geometry

Course Instructor

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Course Website

All updates, including schedule, homework assignments, and references, will be posted on the course website,

`http://math.stonybrook.edu/~azinger/mat562`.

Please visit this website regularly.

Prerequisites

This course is limited to the PhD students in mathematics who have passed their comps. All others must obtain permission from the instructor before registering for this course. Co-enrolling in MAT 545 (complex geometry) and MAT 541 (algebraic topology) would nicely complement this course. Feel free to contact the instructor with any questions.

Grading

Your grade will be based on class participation and light homework assignments *if you have not passed the orals yet* (roughly two problems every two weeks); see the next page for more details.

Readings

The main textbook will be *Introduction to Symplectic Topology*, 3rd ed., by Dusa McDuff and Dietmar Salamon, colloquially known as the *baby McDuff-Salamon* (as opposed to their mammoth J-holomorphic curves book). Unfortunately, the authors use the negative of the standard definition of the Lie bracket of two vector fields on a smooth manifold (see footnote on p23 and the long remark on p98), the notation $\mathrm{d}f(x)v$ for the differential of smooth map f at a point x applied to the tangent vector v (instead of $\mathrm{d}_x f(x)$), and no commas (which oftentimes would have helped with the readability). Many of the same topics (especially the more foundational ones) are covered in a somewhat lighter way in *Lectures on Symplectic Geometry* by Ana Cannas da Silva. We will also cover some of the readings listed on the next page and/or some other papers. You should acquire the main book, but copies of the supplementary readings will be provided as needed.

About the Course

The mathematical field of symplectic geometry and topology arose in the 1950s as a formalization of the equations of motions of classical physics into the study of Hamiltonian flows on symplectic manifolds, now the subfield of SG/ST called Hamiltonian dynamics. Since then, this field has greatly expanded to include such subfields as the geography of symplectic manifolds, i.e. what manifolds could potentially be made symplectic, and the theory of pseudoholomorphic curves, which underpins the connections of SG/ST to string and algebraic geometry. This course will focus on the very foundations of SG/ST that are generally relevant throughout this field. These include the local structure of symplectic manifolds and submanifolds (Darboux Theorem), almost complex structures, some restrictions on the topology of symplectic manifolds, and various constructions of symplectic manifolds (symplectic reduction, cut, and sum).

The *Introduction* and *Chapter 1* of the main book motivate the field of SG/ST and translate the equations of motions of classical physics into the mathematical language of this field; you should read them on your own. *Chapter 2* covers background linear algebra and establishes linear specializations of some of the remarkable SG/ST results established in greater generality later in the book and/or elsewhere in the literature. We will cover the foundational *Chapters 3* and *4* thoroughly, followed by at least parts of the more specialized *Chapters 5-7* and/or related topics from various papers, such as

- *Symplectic toric manifolds*, Section 2 in 1206.2703;
- *Symplectic cut*, Section 1 MR1338784;
- *Symplectic sums and exotic symplectic fourfolds*, Sections 3-6 in MR1356781;
- *Construction of symplectic divisors*, Section 7.4 and MR1438190.

Homework Assignments

There are exercises dispersed throughout the main textbook. These are generally directly related to the text and are thus not very hard. Two or so of these and related exercises will be assigned every two weeks or so. However, you should figure out all (or at least most) of the exercises for yourself. When writing solutions to the assigned exercises, you should take the statements of all preceding exercises as given. Feel free to discuss any of the exercises with anyone else, but do write your own solutions.

You should also read (and study in detail) every section of the book (as well as additional readings) covered in class. The book is thoroughly written, so this should not be too hard.

This is an intermediate grad course, and the formal requirements are fairly light. However, the more effort you put into this course, the more you are likely to benefit from it.