Jim Simons, the Mathematician

from Blaine Lawson

This is an informal note to the Mathematics Department about Jim Simons, who was my friend and collaborator for many years. Since his passing many articles have been written, even one at Stony Brook, which discuss his financial wizardry and his support for many important causes, but which say essentially nothing about him as a mathematician. So this is a brief survey of his work and its amazing influence in both mathematics and physics. Many of you may know much of this, but I thought it would be nice to look back. Longer, more detailed comments from many people will appear later.

Jim Simons was one of the great geometers of the second half of the twentieth century. His work won him the Veblen Prize, one of the highest honors in the field. To his great surprise one of his papers had enormous implications in areas of physics. The Chern-Simons invariant appears in the Lagrangians in many important models. In fact, the number of citations of this work in the physics literature averages FOUR A DAY.

In a story that not so many people know, there was a seminar in roughly 1972 at Stony Brook, run by Frank Yang with Jim Simons as a speaker, which had enormous implications for research in mathematics and physics. It transformed what had been (at the level of research) two essentially distinct subjects, to one of real collaboration at the highest levels. I will talk about this below.

In 1968 Jim was invited to Stony Brook by President Toll to build a mathematics department in this essentially new university. It was not an easy task. Research mathematicians usually prefer places with outstanding faculties. Nevertheless, Jim made the math department at Stony Brook a center of worldwide respect. The group in differential geometry that he built here has been at the core of the field for decades.

Jim was always there for the people and the institutions he cared for. This was certainly true of Stony Brook University. Even when he turned his attention to finance, Jim served essentially continuously in offices and on boards. He wanted Stony Brook to be a major center of research, which it has become.

JIM'S WORK IN DIFFERENTIAL GEOMETRY

Holonomy. Jim's thesis [Ann. of Math., 1962], written at Berkeley under Bertram Kostant, gave an important direct proof of Berger's classification of holonomy groups of riemannian manifolds. Holonomy is the group of transformations given by parallel translation around loops based at a fixed point. Jim showed that the holonomy group always acts transitively on the unit sphere in the tangent space at that point (except for the symmetric spaces of rank ≥ 2). This was an intrinsic argument which really gave insight into the geometry. It also established Berger's theorem without using the classification results in the theory of Lie groups.

Minimal Varieties. The subject of minimal submanifolds has a long history in mathematics with major contributions by Schwarz, Weierstrass, Jesse Douglas (winning the first Field's medal), and leading in the 50's and 60's to the work of many people in Geometric Measure Theory. In particular there were fundamental results of Federer-Fleming, De Giorgi, Almgren, Allard, and many others. Solutions of the Plateau Problem were obtained in great generality, and using tangent cones there was a dimension-reduction scheme, which Almgren was able to use to prove regularity for minimal 3-folds in dimension 4.

In a revolutionary paper [Ann. of Math., 1968] Jim Simons established a fundamental equation and extended this regularity result for hypersurfaces up to 6-folds in dimension 7. He also established the Bernstein Conjecture, that any solution $u : \mathbb{R}^{n-1} \to \mathbb{R}^n$ of the minimal surface equation must be a linear function, for $n \leq 8$.

Jim then produced, in the same paper, a 7-dimensional cone in $\mathbb{R}^8$ for which the volume was stable (second derivative of volume ≥ 0), and he conjectured that this was absolutely volume minimizing. It was the cone

$$\text{Cone}\left(S^3(1) \times S^3(1)\right) \;=\; \{(x, y) \in \mathbb{R}^4 \times \mathbb{R}^4 : |x| = |y|\}.$$

Not long after, Bombieri, Guisti, and Miranda showed that indeed this cone is absolutely volume minimizing, so there is *no interior regularity* for hypersurfaces in dimensions ≥ 8, and also there are counterexamples to the Bernstein Conjecture in all dimensions ≥ 9.

This was absolutely surprising!. Much work followed, and the field had a totally new character.

Projective Cycles. In [Ann. of Math., 1973] Jim Simons and I proved, among other things, that every closed rectifiable current in complex projective space which is volume stable, is a positive algebraic cycle.

Chern-Simons Invariants and Differential Characters. The history of this work is interesting. At a certain point, Jim was trying to find a combinatorial formula for the first Pontryagin class $p_1(M)$ of a compact 4-manifold. Hirzebruch had shown that for any riemannian metric on M, $p_1(M)$ is given by integration of a specific polynomial in the curvature tensor. Jim took a triangulation of M and looked at a family of metrics converging down to a singular one which was flat on all the faces of the triangulation. He then started integrating by parts, planning to get down to the 2-skeleton. However, there was a certain term on the 3-skeleton, that he could not get rid of. In fact, it seemed that whatever he did it was essentially the same. He went to bed, tired and somewhat frustrated. When he awoke, he realized it DID NOT CHANGE! This led him to define a certain invariant for 3-manifolds, and he wrote to Chern about it. Chern said it should generalize to all dimensions, and together they showed that this was true [Ann. of Math., 1974]. These invariants had many applications, for example new sophisticated obstructions to conformal immersions.

As I have mentioned above, Chern-Simons forms play a central role in many areas of physics, for example in theories considered by E. Witten, C. Vafa, N. Nekrasov, and many others.

This work led to a new and deep theory of *differential characters* which was done in a long collaboration with Jeff Cheeger, and had many surprising applications. The set of results with Chern and Cheeger turned out to be an absolutely deep contribution to mathematics. In a separate work Cheeger showed that there exists a ring structure on the characters, which gave many more applications. Reese Harvey and I showed that these invariants satisfy Poincaré-Pontryagin duality, and they occur in many different contexts, in analysis and elsewhere. These characters are also related to the Atiyah-Patodi-Singer index theorem for manifolds with boundary.

Together with Dennis Sullivan, Simons showed that the characters in this general context are axiomatically characterized. This started over a decade of work between Dennis and Jim. They have results about analogues of differential characters associated with any homology theory with geometric cycles. They also showed that the space of bundles with connection on a manifold, up to a certain equivalence, gives a many faceted theory. Their paper in 2017, Jim's last mathematical contribution, is rich with new ideas.

Stable Yang-Mills Fields. Jim showed that any Yang-Mills field on the sphere S^n, $n \geq 5$, is unstable as a critical point of the Yang-Mills functional. Using this J.-P. Bourguignon proved that on S^4 a stable field is actually self-dual (or anti-self-dual) and therefore an absolute minimizer.

THE YANG-SIMONS SEMINAR

In the early 1970's Frank Yang had a Math-Physics seminar at ITP, based on the idea that people in these two fields should have more communication. The rule for the sessions was: If a mathematician is talking, only physicists can ask questions, and conversely if a physicist is talking, only mathematicians can ask questions.

Well, Jim Simons was invited to talk in this seminar, and the most incredible thing happened. They found out that:

Simons, who was working on connections on principal bundles and

Yang, who was working on gauge field theories

were

working on the SAME THING!

It was astounding! Something called the Aharonov-Bohm effect in physics, could be explained simply using holonomy of a flat connection.

The news went like lightning. Is Singer found out (probably from Jim). Michael Atiyah found out (probably from Is), and from these two, many many mathematicians became involved.

At the same time Yang drew up a long dictionary relating terms and theorems in geometry to terms and results in physics.

Atiyah, Singer and Hitchin spent many years learning modern physics and proving results in this interface. There was a year at the Institute for Advanced Study for mathematicians to *really* learn particle physics. At the same time, physicists began proving deep results in algebraic geometry. People like Witten, Vafa, Nekrasov, Douglas, and others, were doing geometry just like the mathematicians. In fact, Witten got the Fields Medal!

My point here is that the Yang-Simons seminar was a true turning point. Physicists and mathematicians were really collaborating on some of the most important problems in both fields.

ABOUT JIM HIMSELF

The most attractive thing about Jim Simons was the way he cared for people. When I first came to Stony Brook, I saw the respect he had for those who worked for him, staff and faculty alike. He wanted everything to grow, and he wanted everyone to benefit. He was very open with mathematical ideas. He was also a lot of fun. A person I knew at Berkeley came to Stony Brook for a while, and he told me parties in Berkeley were nothing compared to those at Stony Brook.

Jim Simons had a very clear mind. He always saw the main point in complicated situations, and would go after it.

People were very important to Jim. He understood what was important to a friend, and the talents that friend had. Because of this, people working for him were happy and productive.

Jim was also extremely loyal. I saw this in many ways – one of which was his devotion to our department and university. As said above, Jim came to Stony Brook with the responsibility of building the mathematics department into one of national respect, which he did. However, over the years, while he was very much immersed in the financial world, he was always concerned about the department, and when things went wrong he was always here to help. The same was true of the university.