

**Stony Brook University
The Graduate School**

Doctoral Defense Announcement

Abstract

One parameter families of Schwarz reflection maps arising from Shabat-Belyi maps

By

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A polynomial which is univalent on a closed disk gives rise to a Schwarz reflection map on the image. One may study the iteration of such maps, which form an anti-holomorphic counterpart to the correspondences studied by S. Bullett, C. Penrose, L. Lomonaco, and others which give matings of rational maps and Kleinian groups. Adapting the straightening theorem of Douady and Hubbard we show that such Schwarz reflection maps with connected non-escaping set are in a dynamical bijection with a certain class of parabolic rational maps, and in particular such reflection maps exist in abundance.

We then turn our attention to special one-parameter families of such Schwarz reflections, when there is a single unconstrained critical value. These families arise when the uniformizing polynomials are Shabat polynomials, and are indexed by rooted, bi-colored, planar embedded trees. We show that the parameter spaces and escape loci are connected and simply connected, and hence the connectedness loci are themselves connected. We give a partial combinatorial description of the connectedness loci. We show, as Douady and Hubbard did for the Mandelbrot set, that there are many renormalizable parameters, giving rise to little copies of Multibrot and Multicorn contained in the connectedness loci. We also use the recent results of Clark-Drach-Kozlovsky-van Strien (generalizing earlier results of Yoccoz and Avila-Kahn-Lyubich-Shen), to show that any parameters which are not renormalizable are combinatorially rigid.

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