SYLLABUS – MAT 324 and MAT 522
REAL ANALYSIS, FALL 2024

Tues. and Thurs. 12:30 AM–1:50 PM
in Earth and Space Sciences 069

PREREQUISITE for this course is MAT 320 with a grade of B or higher.

This is a course in measure theory and integration. The classical Riemann integral works very well for continuous functions, but is insufficient for much of modern analysis and probability theory.

A general theory of integration on a space X is based on knowing the measure or **size** $\mu(A)$ of subsets $A \subset X$. Think of $\mu(A)$ as the integral of the simple function

$$\chi_A(x) \quad = \quad 1 \text{ if } x \in A \text{ and } 0 \text{ if } x \notin A.$$

One can then try to define the integral by limits of *step functions* $f = \sum_k c_k \chi_{A_k}$ for constants c_k's where the A_k are disjoint. However, it turns out that one must be careful – a naive approach does not work. However, with the right understanding, there is a wonderful theory of Lebesgue measure and Lebesgue integration on $\mathbf{R}^n$ with many important results.

Furthermore, this theory is easily generalized to a very wide context. This will be the main content of the course. We will then provide introductions to other areas of analysis where measure theory and integration are important – including the theory of Banach spaces, Hilbert spaces, and the Fourier transform.

INSTRUCTOR: Blaine Lawson

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Office Hours: Tu.-Thu 2:00 - 3:30 PM, or make an arrangement for another time. The best way to contact me is by e-mail, which I read often.

GRADER: Matthew Huynh

E-mail: Matthew.Huynh@stonybrook.edu

Office hours: W 2:30pm-3:30pm

MLC Hours: Math Learning Center Hours:
(in Math Tower S-235 or online)
Wed. 10:00am-11:00am by ZOOM
Wed. 12:00pm-1:00pm

HOMEWORK. Homework will be due at the beginning of class each Tuesday.

EXAMS. There will be a midterm on Oct. 29th.

The final exam is on Dec. 17, 11:15 AM to 1:45 PM.

COURSE GRADING. Homework will count 25%, the midterm 30% and the final 45%.

TEXTBOOK.

Title: **Measure, Integration and Real Analysis**,
Author: Sheldon Axler
Graduate Texts in Mathematics No. 282
Springer Open
ISBN   978-3-030-33142-9   ISBN   978-3-030-33143-6   (e-Book)
ISSN 0072-5285   ISSN 2197-5612 (electronic)

The electronic version of this book is FREE.
It can be downloaded at: https://measure.axler.net
or you can ask me for a copy.

OTHER RESOURCE BOOKS:

1.   *Measure, Integral and Probability* by M. Capinski and E. Kopp, from the Springer Undergraduate Mathematics Series.

2.   *Real Analysis*, by H. Royden and P. Fitzpatrick, Pearson Modern Classics, Pearson Press. (A more advanced book which is very well written.)

Disability Support Services: If you have a physical, psychological, medical, or learning disability that may affect your course work, please contact Disability Support Services (DSS) office: ECC (Educational Communications Center) Building, room 128, telephone (631) 632-6748/TDD. DSS will determine with you what accommodations are necessary and appropriate. Arrangements should be made early in the semester (before the first exam) so that your needs can be accommodated. All information and documentation of disability is confidential. Students requiring emergency evacuation are encouraged to discuss their needs with their professors and DSS. For procedures and information, go to the following web site http://www.ehs.sunysb.edu and search Fire safety and Evacuation and Disabilities.

Academic Integrity: Each student must pursue his or her academic goals honestly and be personally accountable for all submitted work. Representing another person's work as your own is always wrong. Faculty are required to report any suspected instance of academic dishonesty to the Academic Judiciary. For more comprehensive information on academic integrity, including categories of academic dishonesty, please refer to the academic judiciary website at http://www.stonybrook.edu/uaa/academicjudiciary/.

Critical Incident Management: Stony Brook University expects students to respect the rights, privileges, and property of other people. Faculty are required to report to the Office of Judicial Affairs any disruptive behavior that interrupts their ability to teach, compromises the safety of the learning environment, and/or inhibits students' ability to learn.

COURSE OUTLINE

Week of Tuesday:

Aug. 27: Prerequisites from MAT 320. The Riemann Integral.
Outer measure and its Properties.

Sept. 3: Shortcomings of Outer Measure.
Borel Subsets and Measurable Functions

Sept. 10: Measures and their Properties.

Sept. 17: Lebesgue Measure.

Sept. 24: Convergence and Approximation Theorems for Measure.

Oct. 1: Integration Theory.

Oct. 8: Integration Theory and Convergence Theorems.

Oct. 15: (No Class on the 15th.) Convergence Theorems

Oct. 22: Lebesgue Measure on $\mathbf{R}^n$.
Review of Past Weeks.

Oct. 29: **Midterm Exam** on October 29th.
L^p-Spaces and Hölder's Inequality on Oct. 31.

Nov. 5: L^p-Spaces and Hölder's Inequality. Banach Spaces.

Nov. 12: Banach Spaces and Hilbert Spaces

Nov. 19: Hilbert Spaces and the Fourier Transform.

Nov. 26: (No Class on Nov. 28th.) The Fourier Transform.

Dec. 3: The Fourier Transform.
Review.

FINAL EXAM: Dec. 17: 11:15 AM to 1:45 PM.

HOMEWORK

This is always due on Tuesday at the **beginning** of class.

Week 1.

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